Predation, Labor Share and Development *

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Abstract

This paper proposes a new mechanism based on the allocation of labor to help understand why differences among countries have remained stable. We formulate a neoclassical growth model in which agents devote time either to produce or to commit predation. Labor share is the key variable which determines in equilibrium the time devoted to each activity: an increase in the labor share raises the incentive to devote time to production and discourages predation. When the elasticity of substitution between labor and capital is lower than one, the labor share rises throughout the transition while the per capita capital is lower than the steady state level. This increase in the labor share reduces the incentive to predate and increases the incentive to work for production. Empirical evidence supports these results: low per capita income countries have larger portions of predation and present lower labor shares. The standard effects of an increase in the productivity are amplified by the indirect effects of productivity on the reallocation of labor from predation to production. Institutional improvements play a key role in reducing predation and increasing the level of per capita income.

Keywords: Predation, Rent-seeking, Labor share, Growth, Development.

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1 Introduction

Differences among countries have remained stable, and in some cases have even increased in recent decades (see Quah 1996, 1997 and Parente and Prescott, 1993). Much effort in current macroeconomic research has been devoted to trying to explain this fact. In this respect, new features, such as the nature and composition of the economic activities have been explored. It is well-known that in economies, resources are devoted to both productive activities (production of goods and services) and to unproductive activities. Unproductive activities entail a group of activities that share the common feature of being profitable, but wasteful: they use resources to generate rents (i.e., income) but not goods (for example, property crime, fraud, begging, corruption, lobbying, rent-seeking, etc.). We will call all these unproductive activities predation from now on. The empirical evidence suggests that the size of the unproductive sector is larger in low income countries. For example, the share of the criminal predatory sector on GDP is 20.7% for Latin America, while it is 6.89% for United States\(^1\). Another example in the literature is from Bourguignon (1999) who finds that the share of property crime on GDP is 0.5% for United States, while it is 1.5% for Latin America.

Similarly, new empirical studies have evidenced that labor shares vary over time and across countries\(^2\). They have found significant differences among countries with the richest countries showing a larger labor share\(^3\)

This paper presents a mechanism that connects the two empirical facts mentioned above: the greater the predation in developing countries the lower share of labor is in these countries. When labor share is low, agents have little incentive to devote time to

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\(^1\)These numbers are calculated using the studies of Anderson (1999) and Londoño and Guerrero (1998) for US and Latin America respectively. See Bethencourt and Perera-Tallo (2010) for details.

\(^2\)See for example Bentolila and Saint-Paul (2003) and Choi and Ríos-Rull (2008). Two such studies focusing on a large number of countries include those of Harrison (2003) and Jones (2003).

\(^3\)Measurement problems are one of the main issues considered in the empirical literature on labor share. Gollin (2002) have pointed out the difficulties to split the income of self-employed between labor and capital income and he proposes several methods to deal with this problem. He uses a small data set of countries at one point in time in which developed countries are over represented. He reaches the conclusion that labor shares do not vary across countries as widely as standard calculations shows. Harrison (2003) using a similar methodology but with a much larger data set, with respect to both the number of countries and number of years, find enormous differences in labor share across countries, as is displayed in Figure 1. This conclusion is also supported by recent empirical evidence presented in studies by Diwan (2001) and Maarek (2010).
Long term labor shares are the time series average for each country in the years available from 1950 to 1997 (data are from Harrison, 2003). Similarly, per capita incomes relative to the world average are calculated as the average of the time series (data are obtained from the Penn World Table 6.2).
labor share results in fewer incentive to devote time to production and more incentives to predation, agents, therefore, devote more time to predation when per capita income is low. Thus, predation declines during the transition to the steady state when the initial per capita capital is lower than the steady state level.

This paper also offers a new explanation to understand why differences in per capita income among countries has remained stable. Conventional wisdom says that differences in productivity is one of the main sources of differences in per capita income, but it is well known that these differences in productivity are not empirically high enough to generate the observed differences in per capita income. In this respect, the proposed model exhibits a mechanism that amplifies the differences in per capita income generated by differences in productivity and thus, helping reconcile the theory with the empirical observation. The mechanism would be as follows: when productivity rises, there is a positive direct effect on production and an indirect effect due to the accumulation of capital (the rise in productivity increases the return on savings and so, the incentives to accumulate more capital). Together with these standard mechanisms, in the current model there is another additional mechanism which amplifies the effect of productivity on per capita income. This new mechanism related to predation and the assumption that elasticity of substitution is smaller than one, works as follows: when the productivity rises, the per capita capital rises and the labor share increases with the per capita capital (due to the assumption of the elasticity of substitution lower than one), reducing the incentive to predate and increasing the portion of labor devoted to production. This increase in the amount of labor devoted to production has three positive effects on the per capita income: i) a direct effect on per capita production; ii) an indirect effect due to the accumulation of capital: when labor rises, it increases the marginal productivity of capital and the incentive to accumulate more capital and; iii) the reduction in the portion of labor devoted to predation implies that the share of the marginal product of capital that goes to savers grows, thus rising the return on savings and promoting the accumulation of capital.

Along the same lines as the literature that emphasizes the role of differences in institu-

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tions to explain differences in per capita income\textsuperscript{6}, we study the effect of an improvement in the quality of the institutions to deter predation. This institutional change is interpreted as a decrease in the productivity of the predation technology which reduces the incentives to predation and so, increases the portion of labor devoted to production. This increase in labor devoted to production not only has a direct positive effect on production, it also encourages the accumulation of capital due to two mechanisms: i) it increases the marginal product of capital and so, the return on savings and; ii) it reduces the portion of payments to capital that goes to predation, increasing the return on savings. Furthermore, when the capital-labor ratio rises, the labor share in the production sector increases (due to the assumption of elasticity of substitution lower than one), and this reinforces the reallocation of labor from predation to production.

There is a large amount of literature devoted to the allocation of labor among productive and unproductive activities (see for example, Murphy, Shleifer, and Vishny, 1991, 1993, Acemoglu, 1995, Schrag and Scotchmer 1993, Grossman and Kim, 1996 and Chassang and Padró 2010). This literature does not deal with accumulation of capital and therefore cannot analyze the interaction between predation and labor share. Some of these papers analyze how predation affects growth, but they do not analyze how growth and capital accumulation affects predation. Grossman and Kim (1996) include a capital accumulation process in their model which depends on predation, but they do not have labor in their model. Thus, they cannot analyze the role of labor share.

This paper is organized as follows. Section 2 develops a model of two sectors: production and predation. Section 3 derives the agents’ decisions and section 4 defines the equilibrium. Section 5 explains how labor share and predation evolve with per capita capital. Section 6 presents the dynamic behavior of the economy. Section 7 analyzes the role of the predation explaining the per capita GDP differences due to a shift in productivity. Section 8 studies the effect of change in the quality of the institutions which reduces the efficiency of the predation technology. The last section, section 9, concludes and Appendix 10 presents the proofs and technical details.

\textsuperscript{6}See Acemoglu, Johnson and Robinson (2005) for a complete survey.
2 The model

Time is continuous with an infinite horizon. The economy is populated with many identical dynasties of homogeneous agents. There is a single good in the economy, which can be used for consumption and investment in physical capital:

\[ Y(t) = C(t) + K(t) + \delta K(t) \]  

where \( Y(t) \) denotes aggregate production, \( C(t) \) denotes aggregate consumption, \( K(t) \) denotes the aggregate capital and \( \delta \in (0, 1) \) denotes the depreciation rate. \( \dot{K}(t) + \delta K(t) \) is the gross investment.

2.1 Technology

The production technology of this good is given by the following production function:

\[ Y(t) = F(K(t), L(t)) \]  

where \( K \) denotes physical capital and \( L \) denotes the labor. The production function, \( F : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+ \), is assumed to be continuous, increasing (in both arguments), strictly quasi-concave and presents constant return on scale. Furthermore, it is twice-continuously differentiable and strictly increasing in \( \mathbb{R}_+^2 \) and; one zero input implies no output, \( F(0, K) = F(K, 0) = F(0, 0) = 0 \) and Inada conditions are satisfied \( \lim_{\kappa \to 0} F'_K(\kappa, 1) = +\infty \) and \( \lim_{\kappa \to +\infty} F'_K(\kappa, 1) = 0 \) for \( \kappa \equiv K/L \). Finally, the production function \( F(\cdot) \) is assumed to exhibit an elasticity of substitution between its inputs of less than one:

\[ \forall \kappa \in \mathbb{R}_+^+ \quad \sigma_f(\kappa) \equiv \frac{\partial \ln(\kappa)}{\partial \ln (\text{RMST}_{L,K}(\kappa, 1))} = \frac{1}{\frac{\partial \text{RMST}_{L,K}(\kappa, 1)}{\partial \kappa}} \frac{\partial \text{RMST}_{L,K}(\kappa, 1)}{\partial \kappa} < 1 \]

where \( \sigma_f(\kappa) \) is the elasticity of substitution between labor and capital (of the production function), which depends on the capital-labor ratio \( \kappa \equiv K/L \). The assumption that the elasticity of substitution is lower than one implies that the labor share increases with the capital-labor ratio. This property will play a key role in our results.
2.2 Preferences

The preferences of a dynasty are given by a time separable, constant elasticity of substitution utility function:

$$\int_0^\infty u(c(t))e^{-\rho t}dt$$  \hspace{1cm} (3)

where $c(t)$ denotes the per capita consumption of dynasty in period $t$ and $\rho > 0$ is the discount rate of the utility function. Agents do not differentiate between the consumption of different members of the dynasty, they only care about their aggregate consumption.

2.3 The predation technology

Each period agents are endowed with 1 unit of time which can be devoted to undertake two types of economic activities: to produce goods $l$ and to undertake predation $l_p$, that is,

$$1 = l(t) + l_p(t)$$  \hspace{1cm} (4)

With regard to predation activities, we understand all the activities which imply use of resources to obtain incomes without generating production. We include property crimes, fraud, corruption, lobbying, etc. Agents income that is obtained through predation is denoted by $\tilde{y}(t)g(l_p)$, where $\tilde{y}(t)$ is the per capita production and $g : \Re_+ \rightarrow [0,1]$ is the fraction of per capita gross production that each agent obtains when devoting time to predation, which depends positively on the amount of time devoted to such activity $l_p$.

We assume that the function $g(.)$ is strictly increasing, strictly concave, continuous and differentiable of second order and $g(0) = 0$, $g(1) < 1$ and $g'(0) \geq 1$.

3 Agents’ decisions

3.1 Households:

The household’ maximization problem is as follows:

$$\max_{\{c(t),l(t),l_p(t),b(t)\}_{t=0}^\infty} \int_0^\infty u(c(t))e^{-\rho t}dt$$

$$\text{p.a. : } b(t) = w(t)l(t) + r(t)b(t) - g(l_p(t))\tilde{y}(t) + g(l_p(t))\tilde{y}(t) - c(t)$$

\begin{align*}
\text{Net Income from the production sector} & \quad \text{Income from predation}
\end{align*}
\[ l(t) + l_p(t) = 1 \]
\[ y(t) = w(t)l(t) + (\delta + r(t))b(t) \]

where \( b(t) \) denotes the assets of the household, \( w(t) \) is the wage per unit of labor, \( r(t) \) the net return on assets and \( y(t) \) is the household’s gross income. The sign “\( \tilde{\ } \)” over a variable means that this variable is a per capita variable of the economy and therefore the household cannot decide on it. Thus, \( \tilde{l}_p \) denotes per capita labor devoted to predation and \( \tilde{y} \) per capita gross income. Income from the production sector is equal to labor income from the production sector \( w(t)l(t) \) plus financial income \( r(t)b(t) \) minus the amount of this income that is predated by other agents in the economy \( g(\tilde{l}_p(t))y(t) \). The other source of income comes from the predation sector which is equal to \( g(l_p(t))\tilde{y}(t) \). The increase of the household’s assets \( b(t) \) is equal to its savings, which is equal to its income (the one from production plus the one from predation) minus consumption \( c(t) \).

The first order conditions for the interior solution imply the following:

\[ w(t) \left[ 1 - g(\tilde{l}_p(t)) \right] = g'_l(l_p(t))\tilde{y}(t) \tag{6} \]
\[ \frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma_u(c(t))} \left[ (r(t) + \delta) \left( 1 - g(\tilde{l}_p(t)) \right) - \delta - \rho \right] \tag{7} \]

where \( \sigma_u(c(t)) = -\frac{u''(c(t))}{u'(c(t))} \) is the elasticity of the marginal utility. The first of the above conditions (equation 6) specifies that the net wage in the production sector after predation should be equal to the marginal payment of predation activities. That is, the marginal payment of the time devoted to each activity should be equal. Equation (7) is the typical Euler equation. The speed at which consumption grows depends positively on return on savings, \( (r(t) + \delta) \left( 1 - g(\tilde{l}_p(t)) \right) - \delta \) and negatively in the patient rate of the household, \( \rho \). Finally, the more concave the utility function (the higher \( \sigma_u(c(t)) \)), the smoother the consumption path.

The following transversality condition should be also satisfied:

\[ \lim_{t \to +\infty} u'(c(t))e^{-\rho t}b(t) = 0 \]

### 3.2 Firms:

Firms maximize profits:

\[ \max_{k,t^d} F(k, t^d) - wt^d - (\delta + r)k \tag{8} \]
where $k$ denotes the per capita capital.

The first order conditions of the above problem are:

\[ F'_k(k, l^d) = f'(\kappa) = (\delta + r) \]
\[ F'_L(k, l^d) = f(\kappa) - f'(\kappa) \kappa = w \]

where $\kappa = k/l^d$. These conditions are well known and say that firms hire a factor until reaching the point at which the marginal productivity of the factor is equal to its price.

### 4 Equilibrium Definition

The equilibrium definition is standard: equilibrium occurs when agents maximize their objective functions and markets clear. Since all households are alike, we may define equilibrium in per capita terms.

**Definition 1** An equilibrium is an allocation \( \{c(t), l(t), l_p(t), b(t), l^d(t), k(t), \tilde{l}_p(t), \tilde{y}(t)\}_{t=0}^{\infty} \) and a vector of prices \( \{w(t), r(t)\}_{t=0}^{\infty} \).

- Households maximize their utility, that is, \( \{c(t), l(t), l_p(t), b(t)\}_{t=0}^{\infty} \) is the solution of the household’s maximization problem (5).
- Firms maximize profits, that is, \( \forall t \ l^d(t), k(t) \) is the solution of the optimization problem of firms (8).
- Capital market clears: \( \forall t \ k(t) = b(t) \)
- Labor market clears: \( \forall t \ l^d(t) = l(t) \)
- Finally, since households are identical, per capita variables coincide with household variables: \( \forall t \ \tilde{l}_p(t) = l_p(t) \) and \( \tilde{y}(t) = w(t)l(t) + (\delta + r(t))b(t) \).

**Definition 2** Steady state equilibrium is an equilibrium in which both the allocation and the prices always remain constant over time.
5 Predation and per capita capital

5.1 Labor share and capital-labor ratio

Let’s denote \( f(\kappa) = F(\kappa, 1) \) as the production per efficient unit of labor, which depends on the capital-labor ratio \( \kappa \equiv K/L \), and the capital share, \( \alpha(\kappa) \equiv \frac{f'(\kappa)\kappa}{f(\kappa)} \). Remember that it was assumed that the elasticity of substitution between labor and capital is less than one:

\[
\sigma f(\kappa) = \frac{\partial}{\partial \kappa} \left( \frac{1}{RMST_{L,K}(\kappa, 1)} \right) = \frac{1}{RMST_{L,K}(\kappa, 1)} \frac{\partial}{\partial \kappa} f(\kappa) = \frac{1 - \alpha(\kappa)}{f(\kappa)} < 1 \quad (9)
\]

It is easy to prove that the assumption that elasticity of substitution between labor and capital is lower than one implies that labor share increases with capital-labor ratio:

\[
\frac{\partial}{\partial \kappa} (1 - \alpha(\kappa)) = \frac{\partial}{\partial \kappa} \left( \frac{1}{f(\kappa)} f'(\kappa) \right) = -\frac{f''(\kappa)\kappa}{f(\kappa)^2} < 0
\]

5.2 Labor devoted to predation and labor share

Using equation (6) and the fact that all household are identical \((\tilde{l}_p = l_p)\), it follows that:

\[
\phi(l_p) = \frac{g'(l_p)(1 - l_p)}{[1 - g(l_p)]} = 1 - \alpha \quad (10)
\]

where \( \phi : [0, 1] \to \mathbb{R}_+ \) is defined as \( \phi(x) = \frac{g'(x)(1-x)}{1-g(x)} \).

Lemma 1 \( \phi(1) = 0 \) and there is \( l_p^{\text{min}} \in [0, 1) \) such that \( \phi(l_p^{\text{min}}) = 1 \) and such that \( \phi(.) \) is strictly decreasing in \( [l_p^{\text{min}}, 1] \) and \( \phi(l_p) > 1 \) when \( l_p < l_p^{\text{min}} \).

Figure 2.a displays function \( \phi(l_p) \). It follows from Lemma 1 and the Implicit Function Theorem that \( l_p \) is a decreasing function of labor share:

\[
\frac{\partial l_p}{\partial (1 - \alpha)} = \frac{1}{\phi'(l_p)} < 0
\]
Obviously, the amount of labor devoted to production is an increasing function of labor share:

$$\frac{\partial l}{\partial (1 - \alpha)} = -\frac{\partial l_p}{\partial (1 - \alpha)} > 0$$

We may summarize this result in the following corollary:

**Corollary 2** The portion of labor devoted to predation $l_p$ is a strictly decreasing function of the labor share, being $l_p = 1$ when $1 - \alpha = 0$ and $l_p = l_p^{\min} \leq 1$ when $1 - \alpha = 1$.

### 5.3 Labor devoted to predation and capital-labor ratio

Since the amount of labor devoted to predation decreases with labor share and labor share increases with the capital-labor ratio, we conclude that the amount of labor devoted to predation decreases with the capital-labor ratio.

**Proposition 3**

The portion of labor devoted to predation at equilibrium $l_p$ is a strictly decreasing function of the capital-labor ratio in production. The portion of labor devoted to production...
at equilibrium \( l \) is a strictly increasing function of the capital-labor ratio in production. At equilibrium \( l_p \in \left( l_{p \min}, 1 \right) \) and \( l \in (0, l_{p \max}) \) where \( l_{p \max} \equiv 1 - l_{p \min} \in (0, 1) \).

Figure 2.b shows that when the capital-labor ratio rises in the production sector (from \( \kappa_1 \) to \( \kappa_2 \)), due to the elasticity of substitution being lower than one, the labor share rises as well (from \( 1 - \alpha_1 \) to \( 1 - \alpha_2 \)). This reduces the household’s incentives to devote time to predation as figure 2.a shows (passing the labor devoted to predation from \( l_{p1} \) to \( l_{p2} \)). Figure 2.c shows that the rise of the capital-labor ratio in the production sector (from \( \kappa_1 \) to \( \kappa_2 \)) generates a drop in the labor devoted to predation (passing from \( l_{p1} \) to \( l_{p2} \)).

From now on we will call \( l_p(\kappa) \) the function that relates the amount of labor devoted to predation in equilibrium with the capital-labor ratio in the production sector, and \( l(\kappa) \) the function that relates the amount of labor devoted to production in equilibrium with the capital-labor ratio in the production sector.

6 Dynamic Behavior

6.1 Dynamic system

It follows from the equilibrium definition that the dynamic behavior of capital is given by the following equation:

\[
\dot{k}(t) = F(k(t), l(\kappa(t))) - c(t) - \delta k(t)
\]

The above equation yields the following accumulation equation of the capital-labor ratio in the production sector:

\[
\dot{\kappa}(t) = \frac{\dot{k}(t)}{l(t)} - \kappa(t) \frac{\dot{l}(t)}{l(t)}
\]

\[
\dot{\kappa}(t) = f((\kappa(t))) - \frac{c(t)}{l(\kappa(t))} - \delta \kappa(t) - \frac{l'(\kappa(t)) \kappa(t)}{l(\kappa(t))} \dot{\kappa}(t)
\]

\[
\dot{\kappa}(t) = \frac{f((\kappa(t))) - \frac{c(t)}{l(\kappa(t))} - \delta \kappa(t)}{1 + \frac{l'(\kappa(t)) \kappa(t)}{l(\kappa(t))}}
\]

It follows from the Euler equation (7) that:

\[
\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma_u(c(t))} \left[ f'(\kappa(t)) [1 - g(l_p(\kappa(t)))] - \delta - \rho \right]
\]
Thus, the dynamic behavior of the economy may be characterized by the following dynamic system:

\[
\dot{\kappa}(t) = \frac{f \left( (\kappa(t)) \right) - \frac{\kappa(t)}{l(\kappa(t))} - \delta \kappa(t)}{1 + \kappa(t) \frac{\nu(\kappa(t))}{l(\kappa(t))}} - c(t) l(\kappa(t)) - \delta \kappa(t) + \kappa(t) l'(\kappa(t)) l(\kappa(t)) \tag{11}
\]

\[
\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma u(c(t))} \left[ f'(\kappa(t)) \left[ 1 - g \left( l_p(\kappa(t)) \right) \right] - \delta - \rho \right] \tag{12}
\]

\[
\lim_{t \to +\infty} u'(c(t)) e^{-\rho t} \kappa(t) l(\kappa(t)) = 0 \tag{13}
\]

It follows from (12) that in order to analyze the dynamic behavior of the economy, it is important to understand the way in which the return on savings evolves with the capital-labor ratio. The following proposition establishes that the return on savings is a decreasing function of the capital-labor ratio in production, as happens in the neoclassical model.

**Proposition 4** The net return on savings (after predation), \( f'(\kappa) \left[ 1 - g \left( l_p(\kappa(t)) \right) \right] - \delta \), is a decreasing function of \( \kappa \) and \( \lim_{\kappa \to 0} f'(\kappa) \left[ 1 - g \left( l_p(\kappa(t)) \right) \right] - \delta = +\infty \) and \( \lim_{\kappa \to +\infty} f'(\kappa) \left[ 1 - g \left( l_p(\kappa(t)) \right) \right] - \delta = -\delta \).

Note that when the capital-labor ratio in the production sector rises, the marginal rate of the capital \( f'(\kappa) \) goes down but the portion of income that goes to factors after predation \( [1 - g \left( l_p(\kappa(t)) \right)] \) goes up. Thus, there are two opposite mechanisms determining the evolution of the return on savings. However, the above proposition establishes that the return on savings always decreases with the capital-labor ratio in spite of the increasing portion of income that goes to factors after predation.\(^7\)

**Corollary 5** There is a unique steady state with positive amount of capital.

Phase diagram in figure 3 shows that the dynamic behavior of the economy is characterized by the typical saddle point dynamic: there is a unique path which converges to the steady state. This means, that given the initial level of per capita capital\(^8\), there is a

\(^7\)If the return on savings were not monotonic multiple equilibria may arise as shown in other papers in the literature, such as Acemoglu (1995) and Schrag and Scotchmer (1993).

\(^8\)Note that the capital-labor ratio in the production sector is an increasing function of per capita capital and vice versa:
\[
\kappa = \frac{k}{l(\kappa)} \Leftrightarrow \kappa l(\kappa) - k = 0 \Rightarrow \frac{\partial \kappa}{\partial k} = \frac{1}{l(\kappa) + \kappa l'(\kappa)} > 0
\]
unique equilibrium path, which converges to the steady state. When the initial amount of per capita capital is lower than the steady state level, the capital-labor ratio, the consumption and the portion of labor devoted to production grows along the equilibrium path, converging to their steady state levels, while the labor devoted to predation goes down. When the amount of per capita capital is larger than the steady state level the opposite happens.

7 Predation as an amplification mechanism of differences in productivity

Many authors have emphasized the key role of differences in productivity to understand differences in per capita income across countries (see See Easterly and Levine, 2001, Hall and Jones, 1999, and Parente and Prescott, 2000). In this section we are going to modify the model to introduce differences in productivity across countries. More precisely, we are going to consider that production depends on a parameter $A$, which is an index of total
factor productivity:

\[ Y(t) = AF(K(t), L(t)) \]

where \( F(K, L) \) satisfies all the assumptions presented in Section 2. Note that this modification of the model does not affect the relationship between the capital-labor ratio in production and labor share. Thus, it does not affect the relationship between capital-labor ratio in production and labor devoted to predation. Therefore, the dynamic system that describes the behavior of the economy (see equations 11 - 13) is as follows:

\[
\kappa(t) = \frac{Af(\kappa(t))) - \frac{c(t)}{l(\kappa(t))} - \delta\kappa(t)}{1 + \kappa(t)\frac{f(\kappa(t))}{l(\kappa(t))}}
\]

\[
\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma^u(c(t))} [Af'(\kappa(t)) [1 - g(l_p(\kappa(t)))] - \delta - \rho]
\]

\[
\lim_{t \to +\infty} u'(c(t))e^{-\rho t}\kappa(t)l(\kappa(t)) = 0
\]

The effect of an increase in the total factor productivity index \( A \) is displayed in Figure 4. Such increase makes the locus \( \kappa(t) = 0 \) go up and the locus \( \dot{c}(t) = 0 \) move to the right. Thus, the capital-labor ratio and the amount of labor devoted to production at the steady state goes up. This involves an increase in per capita income \( y^{ss} = Af(\kappa^{ss})l(\kappa^{ss}) \) at the steady state. The fact that there is predation in the model and that the labor devoted to predation falls with the capital-labor ratio amplifies the effect of the rise in productivity on per capita income at the steady state due to three mechanisms:

1. The rise in productivity increases the return on savings, thus raising the incentive to accumulate capital (see Euler equation 15). When capital grows, labor devoted to predation falls, reducing the portion of income that goes to predation \( g(l_p(\kappa(t))) \), and this increases the return on savings producing an amplification effect on the increase of capital due to the rise in productivity.

2. Furthermore, the return on savings not only increases due to the fall in the part of income that goes to predation. An additional amplification effect exists because when capital increases, owing to the rise in productivity, the portion of labor devoted to production increases, and this implies an increase in the marginal return on capital and the return on saving amplifying further the effect of productivity on per capita capital.
3. Finally, the amplification effect on the per capita income is not only due to the amplification effect on the per capita capital. There is an additional direct effect of the increase in the amount of labor devoted to production on the per capita income.

![Diagram](image)

**Figure 4: Effect of an increase in the productivity**

Figure 5 shows the tree mechanism described above. Figure 5.a splits the effect of the rise in productivity on the capital-labor ratio $\kappa$ in two parts: i) the standard effect, indicated with number 1, which is the effect of an increase in productivity on the capital-labor ratio, keeping the portion of labor devoted to production, and therefore, the portion of income that is predated constant (standard effect). ii) The amplification effect on the capital-labor ratio $\kappa$ due to predation, indicated by number 2, which is equal to the increase of the capital-labor rate due to the reduction in the portion of the payment of capital that goes to predation. Thus, number 2 in figure 5.a displays the effect of the mechanism 1 shown above: the reduction in labor devoted to predation, reduces the portion of the payment of capital that goes to predation, increasing the return on savings.
and the incentive to save, promoting the accumulation of capital.

Figure 5.b splits the effect of an increase in the productivity on per capita capital in three: i) number 3 displays the standard effect that occurs when the portion of labor devoted to production and predation remain constant (at the initial level \( l(\kappa_{1}^{**}) \) and \( l_{p}(\kappa_{1}^{**}) \)). ii) Number 4 displays the amplification effect due to the reduction in the portion of the payment to capital that goes to predation, which came from 5.a. This effect is referred to in mechanism 1 shown above. iii) Number 5 displays mechanism 2 also explained above. When capital increases in response to an increase in productivity, the labor share rises as well, increasing agents’ incentives to devote a larger portion of their labor to production, raising the marginal productivity of capital and the return on savings, and fostering the accumulation of capital.

Figure 5: Amplification effect of an increase in productivity

Figure 5.c splits the effect of an increase in productivity on per capita income in three: i) standard effect (number 6). ii) The amplification effects of predation on per
capita income due to the increase in incentives to accumulate more capital, which are indexed by number 7 and are referred to in mechanisms 1 and 2 explained above. iii) Finally, number 8 displays the effect on per capita income of mechanism 3: the increase of productivity implies more capital accumulation which increases labor share, encouraging agents to devote more time to production, which has a positive direct effect on production.

It is shown in appendix 10.4 that the effect of the increase in the productivity on per capita income may be split in two parts: the standard effect and the amplification effect due to predation (due to the three mechanism described above), this is,

\[
\frac{\partial y^{ss}}{\partial A} = \frac{A}{y^{ss}} \left[ 1 + \frac{\alpha(k^{ss})\sigma f(k^{ss})}{1 - \alpha(k^{ss})} \right] + \frac{\alpha(k^{ss}) \left( 1 - \sigma f(k^{ss}) \right)}{1 - \alpha(k^{ss})} \left[ \frac{1 + \alpha(k^{ss})\sigma f(k^{ss})}{-g''(l_p(k^{ss}))(k^{ss}) + \alpha(k^{ss})\sigma f(k^{ss})} \right] \]

Note that the amplification effect only occurs when the elasticity of substitution is lower than one. This assumption plays a key role in the amplification effect, since the increase in capital only raises the labor share and the incentives to devote more labor to production when the elasticity of substitution is lower than one.

8 Institutional quality

Many authors have shown the empirical relevance of differences in institutions to explain differences in per capita income (see Acemoglu, Johnson and Robinson (2005) for a complete survey). In this section, we capture this empirical fact by modifying the production function of the predation sector that is now going to depend negatively on an index of institutional quality denoted by $\Gamma \in \mathbb{R}_+$. Thus, an increase in the index of institutional quality reduces the productivity of the predation sector, discouraging the use of labor in that sector and encouraging the use of labor in production. To be more precise, we assume that the amount of per capita gross income that each agent obtains when devoting time to predation is a function $g : \mathbb{R}_+^2 \to [0, 1]$ which is continuous and differentiable of second order, strictly increasing strictly concave in its first argument, that is, $g'_l(l_p, \Gamma) > 0$, $g''_l(l_p, \Gamma) < 0$, $g(0, \Gamma) = 0$, $g(1, \Gamma) < 1$ and $g'_l(0, \Gamma) \geq 1$. Furthermore, we assume that $\forall l_p > 0: g'_l(l_p, \Gamma) < 0$ and $g''_l(l_p, \Gamma) < 0$. 

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Proposition 6 The portion of labor devoted to predation at equilibrium \( l_p \) is a strictly decreasing function of the index of institutional quality \( \Gamma \).

An increase in the index of institutional quality reduces the productivity of the predation technology and, therefore, the incentive to devote time to such activity.

The dynamic behavior of the economy may be characterized by the following dynamic system (see dynamic system 11-13):

\[
\dot{\kappa}(t) = \frac{f((\kappa(t))) [1 - g(\kappa(t), \Gamma, \Gamma)] - \frac{c(t)}{r(\kappa(t), \Gamma)} - \delta \kappa(t)}{1 + \kappa(t) \frac{V(\kappa(t), \Gamma)}{r(\kappa(t), \Gamma)}}
\]  
(17)

\[
\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma u(c(t))} \left[ f'(\kappa(t)) [1 - g(\kappa(t), \Gamma, \Gamma)] - \delta - \rho \right] \quad (18)
\]

\[
\lim_{t \to +\infty} u'(c(t)) e^{-\rho t} \kappa(t) l(\kappa(t), \Gamma) = 0 \quad (19)
\]

Figure 6 shows the effect of an increase in the index of institutional quality \( \Gamma \). Such increases makes the locus \( \dot{\kappa}(t) = 0 \) to go up and the locus \( \dot{c}(t) = 0 \) to move right. Thus, the capital-labor ratio and the amount of labor devoted to production go up at the steady state. Furthermore, the amount of labor devoted to predation relative to the capital-labor ratio goes down. Thus, an improvement in institutions reduces incentives to predation, increases the portion of labor devoted to production and the portion of marginal productivity of capital that goes to savers, thus, fostering capital accumulation.

The effect of the increase in \( \Gamma \) on per capita income is as follows (see appendix 10.6):

\[
\frac{\partial y^{ss}}{\partial \Gamma} = \frac{\alpha(\kappa^{ss})}{{\sigma}'(\kappa^{ss})} [\varepsilon_{p}^{1-\sigma} \alpha(\kappa^{ss}) + (1 - \alpha(\kappa^{ss})) \varepsilon_{p}^{1}\alpha(\kappa^{ss}) + \varepsilon_{p}^{1} \alpha(\kappa^{ss}) + \frac{\varepsilon_{p}^{1}\alpha(\kappa^{ss})}{1 - \alpha(\kappa^{ss})}] \left( \frac{-\varepsilon_{p}^{1} \alpha(\kappa^{ss})}{\varepsilon_{p}^{1} \alpha(\kappa^{ss}) + \varepsilon_{p}^{1} \alpha(\kappa^{ss})} \right) \left( 1 + \frac{\varepsilon_{p}^{1} \alpha(\kappa^{ss})}{\varepsilon_{p}^{1} \alpha(\kappa^{ss}) + \varepsilon_{p}^{1} \alpha(\kappa^{ss})} \right)
\]

where \( \varepsilon_{p}^{1-\sigma}(\kappa, \Gamma) = \frac{\sigma'(\kappa_p(\kappa, \Gamma), \Gamma) \Gamma}{1 - g(\kappa_p(\kappa, \Gamma), \Gamma)} > 0 \) is the elasticity of the fraction of income that goes to production factors with respect to \( \Gamma \), \( \varepsilon_{p}^{1}(\kappa, \Gamma) = \frac{\partial(c, \Gamma)}{\partial \Gamma} \frac{r}{r(\kappa, \Gamma)} \) is the elasticity of labor with respect to \( \Gamma \). Note that the last two lines of the above expression are multiplied by \( (1 - \sigma f(\kappa^{ss})) \). Thus, these two lines represent an amplification effect of the improvement in \( \Gamma \) on per capita income due to the fact that the increase of the capital-labor ratio
reduces the incentives to predation due to the increase in the labor share, which occurs when the elasticity of substitution is lower than one.

Figure 6: Effect of an improvement in institutions

9 Conclusion

This paper has presented a neoclassical growth model with predation in which the elasticity of substitution between labor and capital is lower than one. This property of the production function implies that labor share rises during the transition when the initial per capita capital is lower than the steady state level. This increase in the labor share implies a reduction in incentives to predation and a reallocation of labor from predation to production during the transition. Thus, this paper not only analyzes how predation affects capital accumulation but also how capital accumulation affects predation and the
resulting feedback process.

We also analyze the amplification effect that predation may have on differences in productivity across countries. Even though, many authors have pointed out differences in productivity as the main source of differences in per capita income, these differences in productivity are not empirically high enough to generate the differences that are observed in per capita income across countries. This paper proposes a mechanism that amplifies the differences in per capita income generated by differences in productivity. When productivity rises, there is a direct effect on production and an indirect effect due to the accumulation of capital: the rise in productivity increases the return on savings and so, the incentives to accumulate more capital. In our model, together with these standard mechanisms, other mechanisms appear that amplify the effect of productivity on per capita income which is related with predation and the assumption of the elasticity of substitution smaller than one. When productivity rises, the per capita capital rises, and this, due to the assumption that the elasticity of substitution is lower than one, implies that the labor share increases, reducing the incentive for predation and increasing the portion of labor devoted to production. This increment in the amount of labor devoted to production has three effect: i) a direct effect on per capita production; ii) an indirect effect due to the accumulation of capital: when labor rises, it increases the marginal productivity of capital and the incentive to accumulate more capital; iii) finally, the reduction in the portion of labor devoted to predation implies that the share of the marginal product of capital that goes to savers increases, rising the return on savings and promoting the accumulation of capital.

Finally, we analyze the effect of an institutional change that reduces the productivity of the predation technology. Such change discourages predation by increasing the portion of labor devoted to production. This increase in the labor devoted to production not only has a direct effect on production, it also encourages the accumulation of capital due to two mechanisms: i) it increases the marginal product of capital and therefore the return on savings; ii) it reduces the portion of the payments to capital that goes to predation, increasing the return on savings. Furthermore, when the capital-labor ratio rises, the labor share in the production sector increase, assuming elasticity of substitution lower than one, and this promotes the reallocation of labor from predation to production even
more.
10 Appendix

10.1 Proof of Lemma 1

It was assumed that \( g(1) < 1 \) which implies \( \phi(1) = \frac{g'(1)(1-\alpha)}{\sigma f(1-\alpha)} = 0 \).

Note that if \( l_p < 1 \) and \( \phi(l_p) \leq 1 \) then

\[
\phi'(l_p) = \frac{g'(l_p)(1-l_p) - g'(l_p) + \phi(l_p)g'(l_p)}{1 - g(l_p)} \leq \frac{g'(l_p)(1-l_p) - g'(l_p) + g(l_p)g'(l_p)}{1 - g(l_p)} = \frac{g'(l_p)(1-l_p)}{1 - g(l_p)} < 0 \tag{20}
\]

By assumption \( g'(0) \geq 1 \) and \( g(0) = 0 \), if \( g'(0) = 1 \), \( \phi(0) = \frac{g'(0)(1-0)}{1 - g(0)} = g'(0) = 1 \) and in this case \( l_p^{\text{min}} = 0 \); if \( g'(0) > 1 \), \( \phi(0) = g'(0) > 1 \), since \( \phi(1) = 0 \) it follows from continuity of \( \phi(.) \) and from (20) that there is a unique \( l_p^{\text{min}} \) such that \( \phi(l_p^{\text{min}}) = 1 \). It follows from (20) that when \( l_p > l_p^{\text{min}} \) then \( \phi(l_p) < 1 \) and \( \phi'(l_p) < 0 \). Finally, note that when \( g'(0) > 1 \) it follows from definition of \( l_p^{\text{min}} \) that \( \forall l_p < l_p^{\text{min}} \) \( g(l_p) > 1 \).

10.2 Proof of Proposition 3

\[
\frac{\partial l_p}{\partial \kappa} = \frac{\partial l_p}{\partial (1-\alpha)} \frac{\partial (1 - \alpha)}{\partial \kappa} = \frac{1}{\phi'(l_p)f'(\kappa)} (1 - \alpha(\kappa)) \left[ \frac{1 - \sigma f(\kappa)}{\sigma f'(\kappa)} \right] < 0 \tag{21}
\]

10.3 Proof of Proposition 4

\[
\frac{\partial}{\partial \kappa} \left[ f'(\kappa) [1 - g(l_p(\kappa))] - \delta \right] = f'(\kappa) [1 - g(l_p(\kappa))] \left[ \frac{f''(\kappa)}{f'(\kappa)} - \frac{g'(l_p(\kappa))}{1 - g(l_p(\kappa))} \frac{f'(\kappa)}{f(\kappa)} \right] = \\
= f'(\kappa) [1 - g(l_p(\kappa))] \left[ \frac{f''(\kappa)}{f'(\kappa)} - \frac{\phi(l_p(\kappa))}{1 - l_p(\kappa)} \phi'(l_p(\kappa)) \frac{f'(\kappa)}{f(\kappa)} (1 - \alpha(\kappa)) \left[ \frac{1 - \sigma f(\kappa)}{\sigma f'(\kappa)} \right] \right] = \\
= f'(\kappa) [1 - g(l_p(\kappa))] \left[ \frac{1 - \alpha(\kappa)}{\sigma f'(\kappa) \kappa} - \frac{\phi(l_p(\kappa))}{1 - l_p(\kappa)} \phi'(l_p(\kappa)) \frac{f'(\kappa) \kappa}{f(\kappa)} (1 - \alpha(\kappa)) \left[ \frac{1 - \sigma f(\kappa)}{\sigma f'(\kappa)} \right] \right] = \\
= f'(\kappa) [1 - g(l_p(\kappa))] \left[ \frac{1}{\sigma f'(\kappa) \kappa} - \frac{\phi(l_p(\kappa))}{1 - l_p(\kappa)} \phi'(l_p(\kappa)) \frac{f'(\kappa) \kappa}{f(\kappa)} (1 - \alpha(\kappa)) \left[ \frac{1 - \sigma f(\kappa)}{\sigma f'(\kappa)} \right] \right] = \\
= f'(\kappa) [1 - g(l_p(\kappa))] \left[ \frac{1}{\sigma f'(\kappa) \kappa} - \frac{\phi(l_p(\kappa))}{1 - l_p(\kappa)} \phi'(l_p(\kappa)) \alpha(\kappa) \left[ \frac{1 - \sigma f(\kappa)}{\sigma f'(\kappa)} \right] \right] \tag{22}
\]
Note that:
\[
\frac{\phi(l_p) \ 1}{1 - l_p} \phi'(l_p) = \frac{g'(l_p)\ [1 - g(l_p)] \ \frac{g'(l_p)(1-l_p)}{g'(l_p)}}{[g'(l_p)\ [1 - g(l_p)]} \ 1 - \frac{1}{1-l_p} + \frac{g'(l_p)}{[1 - g(l_p)]} = \frac{1}{\frac{g'(l_p)(1-l_p)}{g'(l_p)}} - 1 + \phi(l_p)
\]

Substituting the above equation in (22) yields:
\[
\frac{\partial [f'(\kappa)[1 - g(l_p(\kappa))]]}{\partial \kappa} - \delta - \rho = \frac{f'(\kappa)[1 - g(l_p(\kappa))](1 - \alpha(\kappa))}{\sigma^f(\kappa)\kappa} \left[ -1 - \frac{\alpha(\kappa) \left[ 1 - \sigma^f(\kappa) \right]}{\frac{g'(l_p(\kappa))(1-l_p(\kappa))}{g'(l_p(\kappa))} - 1 + \phi(l_p(\kappa))} \right] = \frac{f'(\kappa)[1 - g(l_p(\kappa))](1 - \alpha(\kappa))}{\sigma^f(\kappa)\kappa} \left[ -1 + \frac{\alpha(\kappa) \left[ 1 - \sigma^f(\kappa) \right]}{\frac{g'(l_p(\kappa))(1-l_p(\kappa))}{g'(l_p(\kappa))} + \alpha(\kappa)} \right] < 0
\]

where in the third equality we use the equilibrium condition (10) and in the last inequality we use the assumption that states that \( g(.) \) is strictly concave (so \( -\frac{g'(l_p(\kappa))(1-l_p(\kappa))}{g'(l_p(\kappa))} > 0 \)).

10.4 Relationship between per capita income and productivity

10.4.1 Standard case:

When \( l(\kappa^{ss}) = 1 \) (the standard case) the effect of an increase in \( A \) over the steady state capital may be obtained by using the Implicit function Theorem over the Euler Equation at the steady state:

\[
f'(\kappa^{ss}) - \frac{\delta + \rho}{A} = 0
\]

\[
\frac{\partial \kappa^{ss} A}{\partial A} = \frac{\sigma^f(\kappa)}{(1 - \alpha(\kappa))}
\]

where we used equation (9). The effect of a change in productivity over the per capita income at the steady state \( y^{ss} = Af(\kappa^{ss}) \) is as follows:

\[
\frac{\partial y^{ss} A}{\partial A} y^{ss} = 1 + \left[ \frac{f'(\kappa^{ss})\kappa^{ss}}{f(\kappa^{ss})} \right] \frac{\partial \kappa^{ss} A}{\partial A} = 1 + \frac{\alpha(\kappa)\sigma^f(\kappa)}{(1 - \alpha(\kappa))}
\]
10.4.2 Predation:

At the steady state the following condition should be satisfied:

\[ f'(\kappa^{ss})[1 - g(l_p(\kappa^{ss}))] - \frac{\delta + \rho}{A} = 0 \quad (26) \]

Using the Implicit Function Theorem:

\[
\frac{\partial \kappa^{ss}}{\partial A} \frac{A}{\kappa^{ss}} = \frac{1}{\left[ \frac{(1-\alpha(\kappa^{ss}))}{\sigma f'(\kappa^{ss})} - \frac{g'(l_p(\kappa^{ss}))(\kappa^{ss})}{1-g(l_p(\kappa^{ss}))} \right] \left[ \frac{\alpha(\kappa^{ss})}{g'(l_p(\kappa^{ss})) + \alpha(\kappa^{ss})} \right] \left[ \frac{1-\sigma f(\kappa^{ss})}{\sigma f'(\kappa^{ss})} \right]} = \]

\[
= \frac{\sigma f(\kappa^{ss})}{(1-\alpha(\kappa^{ss}))} \left[ 1 - \frac{\alpha(\kappa^{ss})}{\left[ \frac{-g'(l_p(\kappa^{ss}))(\kappa^{ss})}{g'(l_p(\kappa^{ss})) + \alpha(\kappa^{ss})} \right] \left[ 1-\sigma f(\kappa^{ss}) \right] + \frac{\alpha(\kappa^{ss})}{\sigma f(\kappa^{ss})} \right] > 0 \]

where we use equations eqs. (9), (10), (21) and (23) in the first equality. It follows from (21) and the definition of \( \phi(.) \) and equation (23) that:

\[
\frac{\partial l}{\partial \kappa} \frac{\kappa}{l} = - \frac{\partial l_p}{\partial \kappa} l = \]

\[
= \frac{g'(l_p(\kappa))(1-l_p(\kappa))}{(1-l_p(\kappa))} \frac{l(\kappa)}{(1-l_p(\kappa))} \left[ \frac{-g'(l_p(\kappa))(1-l_p(\kappa))}{g'(l_p(\kappa))} + 1 - \phi(l_p) \right] f'(\kappa) \frac{1 - \alpha(\kappa)}{f(\kappa)} \left[ \frac{1-\sigma f(\kappa)}{\sigma f'(\kappa)} \right] = \]

\[
= \frac{\alpha(\kappa)}{g'(l_p(\kappa)) + \alpha(\kappa)} \frac{1 - \sigma f(\kappa)}{\sigma f'(\kappa)} > 0 \quad (27) \]

where in the last equality we used equation (10) and the definition of \( \alpha(\kappa) \) and the equation \( l + l_p = 1 \). We turn now to analyze the effect of change in productivity over the per capita income \( y^{ss} = Af(\kappa^{ss})l(\kappa^{ss}) \):

\[
\frac{\partial y^{ss}}{\partial A} \frac{A}{y^{ss}} = 1 + \left[ f'(\kappa^{ss})\frac{\kappa^{ss}}{f(\kappa^{ss})} + \frac{l'(\kappa^{ss})\kappa^{ss}}{l(\kappa^{ss})} \right] \frac{\partial \kappa^{ss}}{\partial A} \frac{A}{\kappa^{ss}} = \]

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\[ 10.5 \text{ Proof of Proposition 6} \]

Using equation (10):

\[
\phi(l_p, \Gamma) = \frac{g'_p(l_p, \Gamma)(1 - l_p)}{1 - g(l_p, \Gamma)} = 1 - \alpha \tag{28}
\]

\[
\phi'_T(l_p, \Gamma) = \phi(l_p, \Gamma) \left[ \frac{g''_p(l_p, \Gamma)}{g'_p(l_p, \Gamma)} + \frac{g'_T(l_p, \Gamma)}{1 - g(l_p, \Gamma)} \right] < 0 \tag{29}
\]

Using Implicit Function Theorem and equation (20):

\[
\frac{\partial l_p}{\partial \Gamma} = -\frac{\phi'_T(l_p, \Gamma)}{\phi''_p(l_p, \Gamma)} < 0
\]

since \( \phi'_p(l_p, \Gamma) < 0 \) by Lemma 1. \( \square \)

**10.6 Effect of \( \Gamma \) on per capita income**

At the steady state the following condition should be satisfied:

\[
f'(\kappa^{ss})[1 - g(l_p, \kappa^{ss}, \Gamma, \Gamma)] - \delta - \rho = 0 \tag{30}
\]

Using the Implicit Function Theorem:

\[
\frac{\partial \kappa^{ss}}{\partial \Gamma} \frac{\Gamma}{\kappa^{ss}} = \frac{g''_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{1 - g(l_p, \kappa^{ss}, \Gamma, \Gamma)} \frac{\partial l_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{\partial \Gamma} - \frac{g'_p(l_p, \kappa^{ss}, \Gamma)}{1 - g(l_p, \kappa^{ss}, \Gamma, \Gamma)} \frac{\partial l_p(l_p, \kappa^{ss}, \Gamma)}{\partial \Gamma} \frac{\Gamma}{\kappa^{ss}}
\]

\[
= \frac{g''_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{1 - g(l_p, \kappa^{ss}, \Gamma, \Gamma)} \frac{\partial l_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{\partial \Gamma} \frac{\Gamma}{\kappa^{ss}} - \frac{g'_p(l_p, \kappa^{ss}, \Gamma)}{1 - g(l_p, \kappa^{ss}, \Gamma, \Gamma)} \frac{\partial l_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{\partial \Gamma} \frac{\Gamma}{\kappa^{ss}}
\]

\[
= \left[ \frac{1 - \alpha(\kappa)}{\sigma'_{f}(\kappa)} \right] - \frac{g'_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{1 - g(l_p, \kappa^{ss}, \Gamma, \Gamma)} \frac{\alpha(\kappa)}{\frac{\partial l_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{\partial \Gamma} \frac{\Gamma}{\kappa^{ss}}} - \frac{1 - \sigma'_{f}(\kappa)}{\sigma'_{f}(\kappa)}
\]

\[
= \frac{\sigma'_{f}(\kappa)}{(1 - \alpha(\kappa))} \left[ \frac{1}{\Gamma^{1 - g}(\kappa^{ss}, \Gamma) + (1 - \alpha(\kappa)) \varepsilon_1^{T}(\kappa^{ss}, \Gamma)} \right] = \frac{1}{1 - \frac{\alpha(\kappa)[1 - \sigma'_{f}(\kappa)]}{\frac{\partial l_p(l_p, \kappa^{ss}, \Gamma, \Gamma)}{\partial \Gamma} \frac{\Gamma}{\kappa^{ss}}} + \alpha(\kappa)}
\]

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\[
\frac{\partial \gamma^{ss}}{\partial \Gamma} = \frac{f'(\kappa^{ss})\kappa^{ss}}{f(\kappa^{ss})} + \frac{\partial l(\kappa^{ss}, \Gamma)}{\partial \kappa} \frac{\kappa^{ss}}{l(\kappa^{ss}, \Gamma)} + \frac{\partial l(\kappa^{ss}, \Gamma)}{\partial \Gamma} \frac{\Gamma}{l(\kappa^{ss}, \Gamma)} = \\
\left[\alpha(\kappa^{ss}) \left(1 - \frac{1}{\kappa^{ss}} \frac{\alpha(\kappa^{ss})}{\sigma(\kappa^{ss})} \frac{1 - \sigma'(\kappa^{ss})}{\sigma'(\kappa^{ss})} \right) \right] + \varepsilon_1^{1-g}(\kappa^{ss}, \Gamma) = \\
\alpha(\kappa^{ss}) \sigma'(\kappa^{ss}) \frac{\partial \gamma^{ss}}{\partial \Gamma} \frac{\gamma^{ss}}{l(\gamma^{ss}, \Gamma)} \left[\frac{\alpha(\kappa^{ss})}{\sigma'(\kappa^{ss})} \left(1 - \frac{1}{\kappa^{ss}} \frac{\alpha(\kappa^{ss})}{\sigma(\kappa^{ss})} \frac{1 - \sigma'(\kappa^{ss})}{\sigma'(\kappa^{ss})} \right) \right] + \varepsilon_1^{1-g}(\kappa^{ss}, \Gamma) + \\
\left(\frac{1}{\kappa^{ss}} \frac{\alpha(\kappa^{ss})}{\sigma(\kappa^{ss})} \frac{1 - \sigma'(\kappa^{ss})}{\sigma'(\kappa^{ss})} \right) \left(1 + \frac{\alpha(\kappa^{ss})}{\sigma'(\kappa^{ss})} \frac{1 - \sigma'(\kappa^{ss})}{\sigma'(\kappa^{ss})} \right) > 0
\]

where \( \varepsilon_1^{1-g}(\kappa, \Gamma) = -\frac{\sigma'}{\sigma'(\kappa^{ss}, \Gamma)} > 0 \) is the elasticity of the fraction of income that goes to production factors with respect to \( \Gamma \), \( \varepsilon_1^{1-g}(\kappa, \Gamma) = \frac{\partial l(\kappa, \Gamma)}{\partial \Gamma} \frac{\Gamma}{l(\kappa, \Gamma)} > 0 \) is the elasticity of labor with respect to \( \Gamma \), which is positive since \( \frac{\partial l(\kappa, \Gamma)}{\partial \Gamma} < 0 \) (see proposition 8.1). Note that in the first equality of equation we use eqs. (10), (21) and the equation \( l + \lambda = 1 \); in the second equality we use eqs. (10) and (23) and; in the third equality we use equation (10).
References


