Transfers, the Terms of Trade and Capital Accumulation

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Abstract:
The static trade literature has concluded that, absent distortions and bystanders, transfer induced movements in the terms of trade cannot be large enough (under Walrasian stability) to produce the transfer paradox. Dynamic one-sector models have argued that a transfer paradox is possible, but have relied upon international capital mobility and movements in the world interest rate rather than commodity markets and prices. In a dynamic two-sector overlapping generations model – which allows for both static and intertemporal terms of trade effects—commodity trade can produce a steady state transfer paradox under Walrasian stability, and without distortions or bystanders. The existence of the paradox is due to the effect of the transfer on world capital accumulation which is shown to always (that is, for any ranking of factor intensities and savings rates) improve the donor's terms of trade. Transfers may also be Pareto-improving in the steady state, and produce paradoxical welfare results along the transition path.

Key Words: Transfer paradox, Pareto-improving transfers, two-sector overlapping generations model

JEL Classification: F11 , F35, F43, O19, O41

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1. Introduction

For nearly one hundred years, the economics literature has contemplated the effect that an income transfer may have on the terms of trade. The longstanding interest in this topic can be accounted for by the variety of issues that involve international redistributions of income and also the evolving context in which such redistributions take place. Early discussions of what became known as the `transfer problem' concerned the effects of post-first-world-war reparations and made intensive use of static trade models. Transfers were modeled as involuntary, with a donor country transferring real income to a recipient country in a two-sector, static environment. In these models, the direct effect of transfers on welfare of both donor and recipient is obvious – the recipient gains and the donor loses. The problematic aspect of transfers is related to the indirect effect on the relative price of the traded goods. If the recipient’s propensity to spend on the donor’s export good exceeds that of the donor, then a transfer will result in an increase in the relative price of the donor’s exportable. If this favorable effect on the donor’s terms of trade is large enough to outweigh the direct effect of the transfer, the donor will be better off (and the recipient worse off). In the literature, this event is referred to as the `transfer paradox’.

Samuelson (1947) argued that the transfer paradox was not possible in this environment when excess demands followed a Walrasian adjustment process. That is, even when the secondary effect of a transfer works against the direct effect, stability requirements preclude it from being of sufficient magnitude to override the direct effect. Subsequent authors have shown that a transfer paradox might occur provided that other distortions are present (see Bhagwati and Brecher (1982), Bhagwati, Brecher and Hatta (1985) and Grinols (1987)) or there is a third economic agent acting as a bystander to the transfer (see Gale (1974), Bhagwati, Brecher and Hatta (1983), Yano (1983) and Jones (1984)). Thus, as far as the static literature is concerned, a
transfer paradox requires at least a three-agent setting or a distortion. Absent these, there is no transfer paradox.

The static literature expanded in several directions (and continues to do so) but in the context of the Latin American debt problem it was recognized that transfers affected saving-- as well as investment-- decisions and thus a dynamic formulation would be more appropriate (see Eaton (1989)). The first attempts to construct a dynamic analysis of the transfer problem made use of a one-sector overlapping generations model with heterogeneous rates of time preference. In this setting, the indirect effects of the transfer are channeled through the international market for capital and the intertemporal terms of trade. Galor and Polemarchakis (1987) and Cremers and Sen (2008) find that permanent, exogenous transfers can produce the transfer paradox in this environment. The basic intuition behind this result is that if, for example, the donor country is characterized by a lower rate of time preference than the recipient country then the transfer will then lessen world capital accumulation and increase the world interest rate. With the lower rate of time preference, the donor will also be a creditor--and the recipient a debtor-- in the international capital market. The rising interest rate thus simultaneously improves welfare of the donor/creditor while worsening the welfare of the recipient/debtor.

Even a cursory look at the existing literature would convince anyone that the two ways of approaching the transfer problem, namely the earlier trade theoretic static literature and the more recent dynamic literature, have very little by way of a common framework. The former emphasizes the effect of transfers on the static terms of trade, ignoring issues of savings and

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1 Subsequent examples of the transfer problem involved foreign aid to developing economies, oil price shocks, and the Latin American debt crisis.
2 See e.g. Yano and Nugent (1999) for the inclusion of nontraded goods, Kemp and Shimomura (2003) for interdependent utility functions and both Majumdar and Mitra (1985) and Turunen-Red and Woodland (1988) for multilateral transfers.
investment altogether. The one-commodity framework of the latter implies that all adjustments arise via the international capital market and therefore, by construction, the only terms of trade that can be addressed is the intertemporal, rather than the static, terms of trade.

The objective of this paper is to build an analytically tractable model which unifies these two literatures and examines whether a dynamic context has relevance to the transfer paradox when only commodities are traded.\(^4\) This is achieved by casting the problem in a two-country, two-sector overlapping generations model with heterogeneous rates of time preference and trade balance. More specifically, permanent, exogenous transfers are considered in the context of an open economy version of Galor (1992), where the two sectors produce consumption and investment goods. Heterogeneous rates of time preferences are critical to the analysis because they generate different propensities to spend on consumption and investment goods (that is, different savings propensities), without which there could be neither a terms of trade effect nor even trade. The overlapping generations model is also indispensable to the argument because it is the only dynamic framework that is consistent with different rates of time preference and steady state diversification in both countries (see Cremers (2001)), the latter of which is a starting assumption in the static analysis.\(^5\) Also, trade balance is assumed to maintain comparability with the static model. This implies that adjustment to the transfer takes place in commodity, rather than asset markets, as was emphasized in the original transfer literature.

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\(^3\) Others have also analyzed transfers in a dynamic context. In an infinite-lived agent setting, see Brock (1996), Yano (1991), and Gombi and Ikeda (2002), and, in a two-period model see Djajic, Lahiri and Raimondos-Moller (1998) and (1999).

\(^4\) This paper has been motivated by a gap in the theoretical literature, and consequently requires a framework that maintains comparability with existing static and dynamic models. It is clear, however, that these environments collectively deviate from reality in several dimensions. Most notably, in a dynamic setting, voluntary (endogenous) transfers are more likely than permanent (exogenous) transfers. Also, considerations such as altruism and tied-aid have potentially important implications for the possibility of a transfer paradox. These issues, however, are beyond the scope of the current paper.
With different savings propensities, transfers change the capital accumulation paths of both countries, and also the world, as in the one-sector analyses. The specific nature of these changes will depend upon whether it is the donor or the recipient of the transfer that has the greater savings propensity. With two-sectors, however, there are additional developments. First, a change in the world capital-accumulation path will now be reflected over time in the relative price for traded goods in addition to the world interest rate; that is, in both the static and the intertemporal terms of trade. As shown below, whether each rises or falls will depend upon the factor intensities of the two industries, as well as on the ranking of savings rates for the donor and recipient. Furthermore, it will also be shown that the discipline of a two-sector general equilibrium yields a relationship between preferences and the pattern of comparative advantage that increases prospects for the transfer paradox relative to the static analysis. More particularly, for any rankings of savings propensities and factor intensities, the combined effect of the transfer on the static terms of trade and the pattern of trade is guaranteed to be in opposition to the direct effect of the transfer.

In addition to formalizing the above remarks, this paper will further explore the conditions under which the indirect effects are sufficient to produce paradoxical welfare results for the donor and recipient countries. Section 2 sets out the model and derives some preliminary results. Section 3 describes the conditions for Walrasian and dynamic stability. Section 4

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5 In the infinitely lived agent model, different rates of time preference across countries imply that at least one country will be specialized in production the steady state (see Stiglitz (1970)).
6 While the effect of the transfer on capital accumulation will depend upon whether the donor or recipient has the greater savings propensity, it is largely irrelevant to the welfare results. That is, we can generate a steady state transfer paradox with either ranking (see Proposition 3). This is convenient in that it is not clear whether we should interpret the high savings country to be the donor or the recipient. On the one hand, growth models generally refer to country with the largest capital-labor ratio as the ‘rich’ country, suggesting that the high savings country would be the most likely donor. However, in the real world, developed countries often have savings rates that are far below their developing country counterparts (for example, the US and China), conversely suggesting that countries with low savings rates could be donors.
considers the effects of steady state transfers on welfare of the donor and recipient countries, both at and away from the golden rule allocation. It is shown that a transfer-induced change in the static terms of trade always work in favor of the transfer paradox, as described above. Also, it is demonstrated that a transfer paradox is indeed possible at the golden rule steady state with Walrasian price-adjustment, and without pre-existing distortions or bystanders, thereby justifying the interest in a dynamic formulation. Away from the golden rule, it is shown that transfers may improve the welfare of both donor and recipient countries. Section 5 describes conditions under which a donor experiences paradoxical welfare effects out of steady state. Section 6 presents an analytical and numerical example and Section 7 concludes.

2. Model

There are two equally sized trading countries that are identical except for their discount factors. Both countries are populated by overlapping generations of two-period- lived agents with logarithmic utility functions. Each economy has constant returns to scale technologies for the production of two goods, a consumption good, $C$, and an investment good, $I$, using capital and labor inputs. At all dates, it is assumed that both countries are diversified in production. All markets are competitive, however, only the two produced goods are traded internationally; therefore, trade is assumed to balance at every date. The relative price of the investment good is denoted by $p$. The world is initially in a free trade steady state. Into this environment (starting from an initial value of zero) a permanent transfer from one country to the other is introduced at $t = 1$. Let the donor country be denoted by $D$ and the recipient country by $R$. 
2.1 Consumption and welfare

During each time period $t$, an equal number of two-period lived agents are born in countries $D$ and $R$. Let $c^y_t$ and $c^o_{t+1}$ respectively denote the consumption of a member of generation $t$ while young and old. While young, each member of generation $t$ will inelastically supply one unit of labor in exchange for the wage, $w_t$, denominated in units of the consumption good. From this wage, residents of country $D$ will immediately transfer $\tau$ units to their counterparts in country $R$. Also while young, residents of both countries purchase the investment good at (relative) price $p_t$ so as to earn a return which finances old age consumption. Each resident of country $j$, $j=D, R$, will maximize utility, given by $U^j(c^y_t,c^o_{t+1}) = c^y_t c^o_{t+1} \beta^j$, where $\beta^j$ denotes the discount factor for residents of country $j$. The consumption choices for this individual are constrained by a lifetime budget constraint, $c^y_t + c^o_{t+1} / \rho_{t+1} = w^j_t$, where

$$w^D_t = w_t - \tau, \quad w^R_t = w_t + \tau, \quad \rho_{t+1} = r_{t+1} / p_t$$

is the rate of return on capital owned from periods $t$ to $t+1$, and $r_{t+1}$ is the rental paid on capital services during period $t+1$ and is denominated in units of the consumption good. The solution to this optimization problem can be described by an individual savings function, $s^j$, where $s^j(w^j_t,\rho_{t+1}) = w^j_t - c^y_t = \sigma^j w^j_t$ and $\sigma^j = \beta^j / (1 + \beta^j)$ denotes the constant savings rate for residents of country $j$. Also, $c^o_{t+1} = \rho_{t+1} s^j(w^j_t,\rho_{t+1}) = \sigma^j w^j_t \rho_{t+1}$.

Optimization by the household gives rise to an indirect utility function which is an increasing function of the appropriately dated wage and return on capital. Let the indirect utility

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7 With this specification of the utility function, the dynamics become scalar, as described in Section 6 of Galor (1992). Recent papers that utilize OLG models with scalar dynamics to examine trade related issues include Cremers (2001) and Sayan (2005).
function for members of generation $t$ in the donor country be denoted by $V_t^D = V^D(w_t^D, \rho_{t+1})$.

Then, the date $t$ welfare effect associated with the transfer is given by

$$dV_t^D = \left[ V_w^D \frac{dw_t^D}{d\tau} + V_{\rho}^D \frac{d\rho_{t+1}}{d\tau} \right] d\tau$$

where $V_w^D = \partial V^D/\partial w_t^D$, $V_{\rho}^D = \partial V^D/\partial \rho_{t+1}$. Furthermore, Roy’s Identity implies that

$$V_{\rho}^D = V_w^D s^D(w_t^D, \rho_{t+1})/\rho_{t+1}$$

so that

$$dV_t^D = V_w^D \left[ -1 + \frac{dw_t^D}{d\tau} + \frac{s^D(w_t^D, \rho_{t+1}) d\rho_{t+1}}{\rho_{t+1}} \right] d\tau$$

(2.1)

Similar steps, imply that the effect of the transfer on the recipient is given by:

$$dV_t^R = V_w^R \left[ 1 + \frac{dw_t^R}{d\tau} + \frac{s^R(w_t^R, \rho_{t+1}) d\rho_{t+1}}{\rho_{t+1}} \right] d\tau$$

(2.2)

2.2 Resources, technologies and production

In this section we present several preliminary derivations related to production. For simplicity, time indexation is omitted from the notation; elsewhere such notation will utilized when needed.

Both goods are produced at each date using neoclassical, constant returns to scale technologies which are identical in the two countries. These are denoted in intensive form by $f_c(k_c)$ and $f_i(k_i)$ for the consumption and investment goods respectively, where $k_j, j = C, I$, is the capital-intensiveness of sector $j$. It is assumed that the $f_j$ are twice continuously differentiable with $f_j(0) = 0$, $f'_j > 0$, $f''_j < 0$, $f_j'(0) = +\infty$ and $f_j'(\infty) = 0$. In addition, it is assumed that there are no factor intensity reversals; that is, equilibrium capital-intensities
maintain a consistent ranking as the factor price ratio varies. Finally, capital is assumed to
depreciate completely in one period.

In both countries and for both goods, price equals unit cost. Goods prices are set in world
markets and therefore are identical in the two countries. Factor prices also equalize across
countries since factor intensity reversals have been ruled out and both countries are diversified
and use identical technologies. Let $a_{LC}$, a function of the factor prices, denote the labor input per
unit of consumption good, and let other $a$’s be similarly defined. Identical technologies and
factor price equalization together ensure that equilibrium values for the $a$’s are also identical
across countries. Thus, in both countries the requirement that price equals unit cost can be
written $a_{LC}w + a_{KC}r = 1$ and $a_{LI}w + a_{KI}r = p$.

Logarithmically differentiating these conditions (see appendix) yields the Stolper-
Samuelson effects,

$$\hat{\eta}_{a} \equiv \frac{\hat{w}}{\hat{p}} = \frac{-\theta_{KC}}{\Delta}, \quad \eta_{rp} \equiv \frac{\hat{r}}{\hat{p}} = \frac{\theta_{LC}}{\Delta}$$

where each $\eta$ denotes the elasticity of the
indexed factor payment with respect to the relative price of the investment good, a ‘^’ denotes
percentage change, and $\theta_{LC}$, for example, is labor’s cost share of the consumption good. That
is, $\theta_{LC} = a_{LC}w$; $\theta_{LI} = a_{LI}w/p$, $\theta_{KC} = a_{KC}r$; and $\theta_{KI} = a_{KI}r/p$. Furthermore, in the absence
of factor intensity reversals, $\Delta = \theta_{LC} - \theta_{LI} = \theta_{KI} - \theta_{KC} > 0(< 0)$ as the consumption good is
respectively labor- or capital-intensive. Thus, $\eta_{wp} < 0 (> 0)$ and $\eta_{rp} > 0 (< 0)$ as the
consumption good is labor (capital)-intensive.

For each economy, the full-employment conditions are given (in per worker terms) by

$$a_{LC}C^j + a_{LI}I^j = 1, \quad a_{KC}C^j + a_{KI}I^j = k^j$$

where $C^j, I^j$ are production of consumption and
investment per worker in country $j$, and $k^j$ is the capital-labor ratio of country $j$, $j = D, R$.  


These equalities imply that \( a_{LC}C + a_{LI}I = 1 \) and \( a_{KC}C + a_{KI}I = k \) where
\[
C = \left[ C^D + C^R \right] / 2 \quad \text{and} \quad I = \left[ I^D + I^R \right] / 2
\]
are world average consumption and investment goods per worker, and \( k = \left[ k^D + k^R \right] / 2 \) is the world capital-labor ratio.

Differentiating the world resource constraints yields the elasticity of supply for the investment good with respect to its price, \( \eta_{Ip} \equiv \left( \hat{I} / \hat{p} \right) \bigg|_{p=0} = \left[ \lambda_{LC}\delta_K + \lambda_{KC}\delta_L \right] / (\Omega \Delta) > 0 \)
where \( \lambda_{LC} \), for example, is the employment share of labor in the consumption industry, \( \delta_L \equiv \lambda_{LC}\theta_{KC}\varepsilon_C + \lambda_{LI}\theta_{KI}\varepsilon_I \), and \( \delta_K \equiv \lambda_{KC}\theta_{LC}\varepsilon_C + \lambda_{KI}\theta_{LI}\varepsilon_I \). Also,
\[
\varepsilon_C = (\hat{a}_{KC} - \hat{a}_{LC}) / (\hat{w} - \hat{r}) \text{ is the elasticity of substitution in the consumption sector; } \varepsilon_I \text{ is similarly defined. These relationships show that price changes affect factor prices and, via the elasticity of substitution, relative factor inputs, and therefore the supply of the investment good.}

Finally, it is useful to note that \( \Omega \equiv \lambda_{LC} - \lambda_{KC} = \lambda_{KI} - \lambda_{LI} > 0 (< 0) \) as the consumption good is respectively labor- or capital-intensive and also that \( \Omega \) and \( \Delta \) are always of the same sign. Also, we have the Rybczynski effect \( \eta_{Ik} \equiv \left( \hat{I} / \hat{k} \right) \bigg|_{p=0} = \lambda_{LC} / \Omega > 0 (< 0) \) as the consumption good is respectively labor- or capital-intensive.

3. Existence and stability

Using Walras’ law, we focus on the market for the investment good. Country \( j \),
\( j = D, R_1 \) maximizes gross domestic product at each date subject to its resource constraints and subsequently supplies \( I^j = I(p_i, k^j) \) units of produced investment goods per worker to the world market at date \( t \). Constant returns to scale implies that the world per capita supply of investment
goods per worker can be expressed by \( I(p_t, k_t) = \frac{[I(p_t, k^D_t) + I(p_t, k^R_t)]}{2} \). Also at date \( t \), let \( k^j_{t+1} \) denote the per worker demand for investment goods to be used in production by country \( j \) at date \( t + 1 \). Further, let \( k_{t+1} = \frac{[k^D_{t+1} + k^R_{t+1}]}{2} \) denote world demand per worker for investment goods at time \( t \). Trade is assumed to be balanced so that \( s^j(w^j_t, \rho_{t+1}) = \sigma^j w^j_t = p_t k^j_{t+1} \) for \( j = D, R \). World equilibrium at date \( t \) requires that the capital-stock evolves according to

\[
I(p_t, k_t) = \frac{[I(p_t, k^D_t) + I(p_t, k^R_t)]}{2} = k_{t+1} = \frac{[k^D_{t+1} + k^R_{t+1}]}{2}
\]

and also that the market for the investment good clears,

\[
\frac{\sigma^D w^D_t}{p_t} + \frac{\sigma^R w^R_t}{p_t} / 2 = I(p_t, k_t)
\]

A steady state equilibrium is a solution to (3.1) and (3.2) for which \( k_{t+1} = k_t = k \) and \( p_t = p \). Initially, the world is in a free trade steady state with \( \tau = 0 \). Together with factor price equalization, these equations also describe the steady state equilibrium of a closed world economy at date \( t \). Existence of such an equilibrium then follows directly from Galor (1992).

Below, we require the equilibrium to satisfy conditions for both Walrasian and dynamic stability. The former describes the price adjustment process and the latter describes conditions under which the equilibrium converges over time to a steady state. It should be recalled that Walrasian stability rules out the transfer paradox in a two-country static model, and is thus of particular importance.\(^8\)

\(^8\) It may be helpful to note that there is a fundamental distinction between the Walrasian stability condition for the two-sector model and its more familiar one-sector counterpart. In a one-sector model, stability is typically defined using a market for financial capital, which cannot be considered separately from the market for physical capital goods. That is, savings in any period is typically identified as the supply of capital and the demand for capital ownership is derived from the marginal productivity of the subsequent period’s capital stock. In this setting, the return on capital ownership is the intertemporal price that clears the market for capital. In a two-sector framework, however, there is instead a market for physical capital (investment) goods that is
Lemma 1  If $\eta_{wp} - 1 - \eta_{lp} < 0$, then the equilibrium is Walrasian stable.

Lemma 2  If the equilibrium is Walrasian stable and $(1 - \eta_{wp})(1 - \eta_{lk}) + \eta_{lp} > 0$, then the steady state is dynamically stable.

The proof for each lemma is found in the appendix. Under either factor intensity assumption $(1 - \eta_{wp})(1 - \eta_{lk})$ is negative whereas $\eta_{lp}$ is positive. Thus, for the dynamic stability condition to be satisfied, $\eta_{lp}$, and hence the elasticities of substitution, $\varepsilon_C$ and $\varepsilon_I$, cannot be too small. Below, it is assumed that the conditions stipulated by the lemmas are always satisfied.

4. Transfers and steady state welfare

This section considers the effects of transfers on steady state welfare in the donor and recipient countries, both at and away from the golden rule allocation.

The envelope theorem implies that $dw/dp + k^j (dr/dp) = I(p,k^j)$ or, equivalently,

$$\frac{dw}{dp} = I(p,k^j) - k^j (dr/dp), \ j = D, R.$$  Trade balance and $\left(\frac{p}{\rho}\right) \left(\frac{dp}{dp}\right) = \left(\frac{p}{\rho}\right) \left(\frac{dr}{dp}\right) - 1$,

further imply that the steady state version of (2.1) can be expressed

$$\frac{dV^D}{d\tau} = V^D_w \left[ -1 + \frac{dp}{d\tau} \left( I(p,k^D) - k^D \left(\frac{dr}{dp}\right) - \left(\frac{p}{dp}\right) \right) \right]$$  (4.1)

separate and distinct from the market for financial capital. In this market, savings constitute the demand for investment goods and there is also a very clearly defined supply of investment goods. The relative price of the investment good is then the appropriate market-clearing price.
Similar steps imply that the steady state version of (2.2) is given by

\[
\frac{dV^R}{d\tau} = V_w^R \left[ 1 + \frac{dp}{d\tau} \left( I(p,k^R) - k^R \right) - k^R \left( \rho - 1 \right) \frac{d\rho}{d\tau} \right]
\]  

(4.2)

The first bracketed term in each of the above equations represents the direct effect of a transfer on welfare, and is thus negative for the donor and positive for the recipient. The remaining terms reflect the indirect effects of the transfer and are channeled through the relative price of the investment good, and the return on capital, respectively. These effects are referred to as the static and intertemporal terms of trade effects.

With transfers, steady state market-clearing (see (3.2)) is given by

\[
\sigma w - D_\sigma \tau = pI(p,k)
\]

where \( \sigma = \left[ \sigma^D + \sigma^R \right] / 2 \) and \( D_\sigma = \left[ \sigma^D - \sigma^R \right] / 2 \). Differentiation then yields

\[
-D_\sigma d\tau/\sigma w = \left[ -\eta_{wp} + 1 + \eta_{ip} \right] \hat{p} + \eta_{ik} \hat{k} = 0
\]  

(4.3)

where the coefficient for \( \hat{p} \) is positive by Walrasian stability (Lemma 1). In steady state, the law of motion for capital is \( \hat{k} = I = \eta_{ik} \hat{k} + \eta_{ip} \hat{p} \), so that \( \hat{k}/\hat{p} = \eta_{ip}/(1 - \eta_{ik}) < (>) 0 \) as the consumption good is, respectively, labor or capital intensive. Substituting into (4.3), and evaluating at the initial value of the transfer, \( \tau = 0 \), gives

\[
\frac{dp}{d\tau} = -\frac{D_\sigma}{\eta_{ik}} \left[ \frac{1 - \eta_{ik}}{(1 - \eta_{wp})(1 - \eta_{ik}) + \eta_{ip}} \right]
\]  

(4.4)

Note that dynamic stability (Lemma 2) implies that the denominator of the bracketed term is positive. The sign of the numerator depends upon factor intensity rankings. When the consumption good is labor-intensive, the numerator is negative and \( sgn dp/d\tau = sgn D_\sigma \). When the consumption good is capital intensive, the numerator is positive and \( sgn dp/d\tau = -sgn D_\sigma \).
The effect of the transfer on steady state welfare depends also on the pattern of trade, or
$I(p, k^j) - k^j$, $j = D, R$. The following, rather intuitive result can be easily verified and is left to
the interested reader,

$$I(p, k^D) - k^D = -\frac{D_\sigma}{\sigma^2} \eta_{wp} k.$$  \hspace{1cm} (4.5)

This equality shows that the donor exports investment goods in two cases: i) when it has a higher
savings rate than the recipient and the consumption good is labor intensive and ii) when it has a
lower savings rate than the recipient and the consumption good is capital intensive. There are
two other cases to be considered, under which the donor instead exports the consumption good.
This equality, together with the corresponding implication for the recipient, asserts that the high-
saving country, regardless of whether it is the donor or the recipient, will always export the
capital-intensive good, whether that good happens to be the consumption or the investment good
and vice versa for the low saving country.

Substituting (4.4), (4.5) and $k^D / k = \alpha^D / \alpha$ into (4.1) now gives

$$\frac{dV_D}{d\tau} = V_w^D \left[-1 + \sqrt{\left(\frac{D_\sigma}{\sigma}\right)^2 \eta_{wp} + \frac{\sigma^D}{\sigma}(\rho - 1) \eta_{rp}} \left(1 - \frac{(1 - \eta_{rk})}{(1 - \eta_{wp})(1 - \eta_{rk}) + \eta_{lp}} \right) \right].$$ \hspace{1cm} (4.6)

This expression and its counterpart for the recipient will reveal the conditions under which the
transfer produces paradoxical welfare effects. A steady state transfer paradox occurs when the
donor gains and the recipient loses, or when, $dV_D / d\tau > 0$ and $dV_R / d\tau < 0$. A Pareto-
 improving transfer occurs when both parties gain, or when $dV_D / d\tau > 0$ and $dV_R / d\tau > 0$.

First, to challenge the results of the static literature, it is assumed (temporarily) that the
world economy is initially at the golden rule capital stock, so that $\rho = 1$. This isolates the static
terms of trade effect while the possibility of a transfer paradox is considered. Propositions 1 and 2 explore the sign and the magnitude of the static terms of trade effect respectively.

**Proposition 1** If \( \rho = 1 \), the static terms of trade effect is always positive for the donor and negative for the recipient; therefore, it always favors the transfer paradox.

**Proof** The static terms of trade effect for the donor is given by

\[
\frac{dp}{d\tau} I(p, k^D) - k^D = \left( \frac{D_\sigma}{\sigma} \right)^2 \frac{\eta_{wp} \left( 1 - \eta_{lk} \right)}{\left( 1 - \eta_{wp} \right) \left( 1 - \eta_{lk} \right) + \eta_{lp}}.
\]

(4.7)

For either factor intensity assumption, \( \eta_{wp} \left( 1 - \eta_{lk} \right) \) is positive. Dynamic stability implies that the denominator is also positive. Thus, (4.7) is positive. Trade balance then implies

\[
\frac{dp}{d\tau} I(p, k^R) - k^R = -\frac{dp}{d\tau} I(p, k^D) - k^D < 0.
\]

Thus, under the assumption of both Walrasian and dynamic stability, the donor’s terms of trade effect is positive regardless of the ranking of savings rates or the factor intensity assumption. This result is at odds with the familiar. In the static formulation of the transfer paradox, the effect of a transfer on the terms of trade is determined entirely by the respective marginal propensities to consume of the donor and recipient. Thus, only when the recipient has a higher propensity to spend on the donor’s export will there be a possibility for the donor’s terms of trade to improve. But, in the static framework, preference attributes need not have any bearing on the pattern of comparative advantage. In our dynamic context, a large relative propensity to save is associated with a relative abundance of capital in the steady state and thus a comparative advantage in whichever good is capital-intensive. In other words, in the dynamic setting, the
ranking of the two countries’ marginal propensities to save is inextricably linked with the steady state pattern of comparative advantage.

Recalling the origin of (4.7), it is helpful to note that (4.4) and (4.5) both have the sign of $D_\sigma$ when the consumption good is labor intensive and both have the sign of $-D_\sigma$ when the consumption good is capital intensive. Thus, for a given factor intensity assumption, the ranking of savings rates determines whether or not the donor exports the investment good and also whether the transfer has increased or decreased the relative price of the investment good, $p$. If, for example, the consumption good is labor intensive and the donor has the higher savings rate, then the donor exports the investment good and the donor’s terms of trade improve. If, under the same factor intensity assumption, the donor instead has the lower savings rate, then the donor imports the investment good and its terms of trade, now $1/p$, also improve. Analogous arguments can be made under the assumption of a capital-intensive consumption good. To sum, the circumstances under which the relative price of capital falls as a result of the transfer are the same circumstances under which the donor becomes a steady state importer, rather than an exporter, of the investment good. This explains why the donor’s terms of trade always improve as a result of a transfer.

Another way to view these results is via (4.3), which provides insight with regard to the difference between static and dynamic frameworks in examining the welfare effects of transfers. To make the comparison precise, note that the static model has two consumption goods; hence, the investment good in our model is to be interpreted as the second of the consumption goods. Also, $\sigma_j$ is the marginal propensity to spend on the second good by the residents of country $j$. If the model were not dynamic, the second term-- reflecting steady state effects on capital accumulation--would be zero. In this case, if the donor had the higher savings rate, a transfer
would imply an unambiguous deterioration in the donor’s terms of trade under Walrasian stability. As the earlier literature suggested, this would reinforce rather than negate the direct effects of the transfer and obviate the possibility of a transfer paradox. In a dynamic model, however, the effects on steady state capital accumulation must also be taken into account and are reflected by a non-zero second term. Thus, for the same scenario just described, a transfer of income to the low-saving country now reduces world steady state capital accumulation. When the investment good is capital intensive, the scarcity thus introduced has a positive influence on the donor’s terms of trade (see(4.4)) and consequently reintroduces the possibility of the transfer paradox.

**Proposition 2** Suppose \( \rho = 1 \). Then, \( \varepsilon_C \) and \( \varepsilon_I \) must together be sufficiently large to ensure dynamic stability but sufficiently small to ensure the transfer paradox.

**Proof** First, if \( \rho = 1 \), then (4.1), (4.2) and trade balance imply that \( \text{sgn} \frac{dV^D}{d\tau} = -\text{sgn} \frac{dV^R}{d\tau} \). It is then sufficient to identify conditions for which \( \frac{dV^D}{d\tau} > 0 \), or equivalently (from (4.7)),

\[
\left( \frac{D_\sigma}{\sigma} \right)^2 \left[ \frac{\eta_{wP} (1 - \eta_{IK})}{(1 - \eta_{wP})(1 - \eta_{IK}) + \eta_{Ip}} \right] > 1 \tag{4.8}
\]

Proposition 1 established that the left hand side of this inequality is always positive under Lemma 1 and Lemma 2. If \( (1 - \eta_{wP})(1 - \eta_{IK}) + \eta_{Ip} \to 0 \), then the left hand side will tend to \( +\infty \), satisfying (4.8). Substituting definitions for the \( \eta \)’s gives

\[
(1 - \eta_{wP})(1 - \eta_{IK}) + \eta_{Ip} = (-\lambda_K\delta_{Kt} + \lambda_L\delta_K + \lambda_K\delta_L)/\Omega \Delta \tag{4.9}
\]

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where $\Omega \Delta > 0$. Thus, substituting for the $\delta$’s, any pair of $\varepsilon$’s that satisfy $	heta_{KI}\lambda_{KC} = \\
\lambda_{LC}\left[\lambda_{KC}\theta_{LC}\varepsilon_{C} + \lambda_{KI}\theta_{LI}\varepsilon_{I}\right] + \lambda_{KC}\left[\lambda_{LC}\theta_{KC}\varepsilon_{C} + \lambda_{LI}\theta_{KI}\varepsilon_{I}\right]$ imply that (4.9) equals zero. Then increasing either or both of the $\varepsilon$’s slightly implies that (4.9) is positive and near zero. If $\varepsilon$’s are increased too much, then (4.9) will deviate significantly from zero and violate (4.8). □

Proposition 2 establishes that the static terms of trade effect alone may in fact be large enough to dominate the direct effect of the transfer under Walrasian stability in a dynamic model. It is interesting to further note that, even with general functional forms, neither a factor intensity assumption nor a ranking of savings propensities are required for this result. To further clarify the requirements on the elasticities of substitution, however, a special case is considered where both industries have a common elasticity of substitution. Under this simplification, it is possible to be precise about the range within which the elasticity is consistent with stability and a transfer paradox.

**Proposition 3** If $\rho = 1$, $\varepsilon_{C} = \varepsilon_{I} = \varepsilon$, and $\varepsilon$ satisfies

$$
\frac{\lambda_{KC}\theta_{KI}}{\lambda_{KC}\theta_{KI} + \lambda_{LC}\theta_{LI}} < \varepsilon < \frac{\lambda_{KC}\theta_{KI} + \lambda_{KC}\theta_{KC}\left(D_{\sigma}/\sigma\right)^2}{\lambda_{KC}\theta_{KI} + \lambda_{LC}\theta_{LI}},
$$

then the steady state is dynamically stable and the transfer paradox obtains.

**Proof** If, as required by Lemma 2, (4.9) is positive and $\varepsilon_{C} = \varepsilon_{I} = \varepsilon$, then

$$
\varepsilon > \lambda_{KC}\theta_{KI}\left[\lambda_{KC}\theta_{KI} + \lambda_{LC}\theta_{LI}\right]^{-1}. \text{ The same substitution in (4.8) gives }

\left(D_{\sigma}/\sigma\right)^2 \lambda_{KC}\theta_{KC}\left[-\theta_{KI}\lambda_{KC} + \varepsilon\left(\lambda_{LC}\theta_{LI} + \lambda_{KC}\theta_{KI}\right)\right]^{-1} > 1. \text{ Rearranging the latter gives the expression on the right hand side of the stated inequality. □}

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Proposition 3 is helpful because it demonstrates that the permissible range for the
elasticity of substitution is indeed nonvacuous provided only that \( D_\sigma \neq 0 \), or, that the donor and
recipient have differing savings rates. Again, the transfer paradox does not depend upon whether
it is the donor or the recipient that has the higher savings rate, nor does it depend upon a
particular factor intensity ranking.

Next, (4.1) and (4.2) are reconsidered when the initial world steady state capital-labor
ratio deviates from the golden rule level; that is, when \( \rho \neq 1 \). This implies that the intertemporal
terms of trade effect is nonzero. Regardless of whether or not the static terms of trade effect is
large enough to deliver the transfer paradox, the intertemporal effect introduces the possibility
that the transfer is Pareto-improving for steady state generations.\(^9\)

**Proposition 4** If \( \rho \neq 1, \varepsilon_C = \varepsilon_I = \varepsilon \) and either \( \text{sgn} \, D_\sigma < 0 \) and \( \rho > 1 \), or \( \text{sgn} \, D_\sigma > 0 \)
and \( \rho < 1 \), then the transfer may be welfare-improving for steady state generations in both the
donor and recipient countries.

**Proof** Recalling (4.6), the intertemporal terms of trade effect is given by

\[-k^j (\rho - 1) \eta_\rho (dp/d\tau) = (\rho - 1) D_\sigma \frac{\sigma^j}{\sigma \left( 1 - \eta_{wp} \right) \left( 1 - \eta_{pk} \right) + \eta_{lp}} \text{ for } j = D, R.\]

Regardless of the factor intensity assumption, \( \text{sgn} \, \eta_\rho (1 - \eta_{lk}) < 0 \). By Lemma 2, \( (1 - \eta_{wp})(1 - \eta_{lk}) + \eta_{lp} > 0 \). If
\( \text{sgn} \, D_\sigma < 0 \) and \( \rho > 1 \), or, \( \text{sgn} \, D_\sigma > 0 \) and \( \rho < 1 \), then the effect is positive for both donor and
recipient. If the conditions of Proposition 3 are satisfied, then the additional positive effect

\(^9\) It is also possible for the transfer to make both donor and recipient worse off. If \( \text{sgn} \, D_\sigma < 0 \)
and \( \rho < 1 \), or, \( \text{sgn} \, D_\sigma > 0 \) and \( \rho > 1 \), then the intertemporal effect is negative for both countries.
implies that overall $dV^D / d\tau$ is unambiguously positive and $dV^R / d\tau$ is of ambiguous sign. If the conditions of Proposition 3 are not satisfied, then the additional positive effect implies that $dV^R / d\tau$ is now unambiguously positive and $dV^D / d\tau$ is of ambiguous sign. In either case, $dV^D / d\tau > 0, dV^R / d\tau > 0$ cannot be ruled out. □

The intertemporal terms of trade effect can be understood in terms of the position of the initial steady state relative to the golden rule and the transfer-induced changes in capital accumulation. When $\text{sgn} \, D^\sigma < 0$ and $\rho > 1$, the donor has a lower savings rate and there is no overaccumulation of world capital in the initial steady state. When $\text{sgn} \, D^\sigma > 0$ and $\rho < 1$, the donor has the higher savings rate and there is overaccumulation. In the first case, the transfer increases the world capital labor ratio and capital accumulation is welfare improving. In the second, the transfer reduces the world capital labor ratio but capital accumulation is welfare worsening. In both cases, the intertemporal terms of trade effect will work to the benefit of both the donor and recipient. 10

5. Transfers and transitional welfare

This section considers the effects of the permanent transfers on the welfare of generations living during the transitional periods. For brevity, the focus will be on the donor country. Noting that $d\psi_t = w_t \eta^\psi \tilde{p}_t$, $d\rho_{t+1} = \rho_{t+1} \tilde{p}_{t+1} - \tilde{p}_t$, and $V^D_w = (\sigma^D w \rho)^{\beta^\sigma}$, (2.1) can be rewritten as

$$dV^D_t = (\sigma^D w \rho)^{\beta^\sigma} \left[ -d\tau + w \eta^\psi \tilde{p}_t + \sigma^D \rho (\eta^\psi \tilde{p}_{t+1} - \tilde{p}_t) \right]$$

(5.1)

10 The transfer paradox remains a possibility away from the golden rule. This happens when, for example, the static terms of trade effect dominates the transfer and the intertemporal effect is positive but not large enough to improve welfare of the recipient.
when evaluated at the initial steady state.

The $t = 1$ version of (4.3), with $\hat{k}_1 = 0$, gives $\hat{p}_1 = D_\sigma / (\sigma w_1)$ where

$$\psi \equiv \eta_{wp} - 1 - \eta_{lp}.$$  The date $t$ market-clearing condition is given by $\sigma w_1 - D_\sigma \tau = p_t I(p_t, k_t)$

which, upon differentiation, yields $-(D_\sigma / \sigma w_1) d\tau = -\psi \hat{p}_t + \eta_{lk} \hat{k}_t$. The world capital-labor evolves according to $\hat{k}_{t+1} = \hat{I}_t = \eta_{lp} \hat{p}_t + \eta_{lk} \hat{k}_t$. From these two expressions it can be shown that

$$\hat{p}_{t+1} = \Theta \hat{p}_t + (1 - \eta_{lk}) \hat{p}_1$$

(5.2)

where $\Theta \equiv \eta_{lk}(\eta_{wp} - 1)\psi^{-1}$.

Substituting $\hat{p}_2$, as given by (5.2), and $\hat{p}_1$ into the $t = 1$ version of (5.1) yields

$$\frac{dY_D^t}{d\tau} = (\sigma^D w_1 p_t)^{\beta p} \left[ \frac{\eta_{wp}}{\sigma^D} + \eta_{lp}(\Theta + 1 - \eta_{lk}) - 1 \right] \frac{\sigma^D D_\sigma}{\sigma \psi} - 1 \right]$$

Hence the transfer increases welfare in the donor country of the generation born in the initial period when the bracketed expression is positive.\(^{11}\) Repeated substitution using (5.2) implies that

$$\hat{p}_{t+1} = \left[ \Theta^t + (1 - \eta_{lk}) \sum_{i=0}^{t-1} \Theta^i \right] \hat{p}_1.$$  Using the date $t$ and $t + 1$ versions of this equality in (5.1)


gives the date $t$ welfare effect in the donor country. The transfer scheme increases welfare for generation $t$, $t \geq 2$, in the donor country if

$$\left[ \Theta^{t-1} \left[ \frac{\eta_{wp}}{\sigma^D} - 1 + \eta_{lp}(1 - \eta_{lk} + \Theta) \right] + (1 - \eta_{lk}) \left[ \frac{\eta_{wp}}{\sigma^D} - 1 + \eta_{lp} \sum_{i=0}^{t-2} \Theta^i \right] \frac{\sigma^D D_\sigma}{\sigma \psi} > 1. \right]$$

(5.3)

This expression will be used, in the numerical analysis below, to determine whether a welfare improvement is possible for the transitional generations in the donor country.

\(^{11}\) In both countries, the welfare of old generations living at the onset of the transfers will be affected by $\hat{p}_1$ only, which may be either positive or negative, depending upon the sign of $\hat{p}_1$ and the factor intensity assumption.
6. An example

This section reexamines the welfare findings described above while adopting a specific production functions for the consumption and investment goods: \( f_C(k_C) = k_C^a \) and \( f_I(k_I) = k_I^b \), \( 0 < a, b < 1 \). The first step is to find analytical expressions the \( \theta \)'s and the \( \lambda \)'s when evaluated at the initial steady state. With these particular technologies, it is immediate that

\[ \varepsilon_C = \varepsilon_I = \varepsilon = 1 \] and cost shares are given by \( \theta_{KC} = a, \theta_{KI} = b, \theta_{LC} = 1 - a, \theta_{LI} = 1 - b \).

Also, the world resource constraints imply that \( \lambda_{LC} = (k_I - k) / (k_I - k_C) \),
\[ \lambda_{LI} = (k - k_C) / (k_I - k_C), \lambda_{KC} = \lambda_{LC}k_C / k, \lambda_{KI} = \lambda_{LI}k_I / k. \] Competitive factor markets can be shown to imply that \( k_C = (\Gamma_C / \Gamma_I)k_I \) and \( k_i = \Gamma_i p^{a-b}, i = C, I \), where

\[ \Gamma_C = \left[ \frac{b}{a} \frac{b}{a-b} \frac{1-b}{(1-a)^{(1-a)/(1-b)}} \right]^{1-\frac{a}{1-b}} \quad \text{and} \quad \Gamma_I = \left[ \frac{a}{a-b} \frac{a}{a-b} \frac{1-b}{(1-a)^{(1-a)/(1-b)}} \right]^{1-\frac{a}{1-b}}. \] By substitution,

\[ \lambda_{LI} = \left[ k - \Gamma_C \frac{1}{p^{a-b}} \right] \frac{1}{(\Gamma_I - \Gamma_C) p^{a-b}} \frac{1}{1-1}. \] World market-clearing imposes a second requirement on \( \lambda_{LI} \), namely that the world per capita production of capital equals world savings,

\[ \lambda_{LCK} = \sigma(1-b)k_I^{\frac{b}{a}}, \] or that \( \lambda_{LI} = \sigma(1-b) \). With the previous equality, it follows that

\[ k = \frac{1}{p^{a-b}} \Gamma(\sigma) \] where \( \Gamma(\sigma) = \Gamma_C - \sigma(1-b)(\Gamma_C - \Gamma_I). \) Thus, at the initial steady state,

\[ \lambda_{LC} = (\Gamma(\sigma) - \Gamma_I)(\Gamma_C - \Gamma_I)^{-1}, \lambda_{LI} = (\Gamma_C - \Gamma(\sigma))(\Gamma_C - \Gamma_I)^{-1}, \]
\[ \lambda_{KC} = (\Gamma_C / \Gamma(\sigma))(\Gamma_C - \Gamma_I)(\Gamma_C - \Gamma_I)^{-1}, \text{ and } \lambda_{KI} = (\Gamma_I / \Gamma(\sigma))(\Gamma_C - \Gamma(\sigma))(\Gamma_C - \Gamma_I)^{-1}. \]

Finally, the initial steady state capital-labor ratio must satisfy market-clearing, \( k = \sigma(1-b)k_I^{\frac{b}{a}} \).
\[\sigma(1 - b)\left(\frac{\Gamma_I}{\Gamma(\sigma)}\right)^b k^b, \] which can be solved for \( k = \left[\sigma(1 - b)\left(\frac{\Gamma_I}{\Gamma(\sigma)}\right)^b\right]^{\frac{1}{1 - b}}. \] This capital-labor ratio implies a world return on capital equal to \( \rho = (a(1 - \sigma) + b\sigma) / (\sigma(1 - a)) \).

With these specifications, dynamic stability (Lemma 2) requires
\[
\left[1 + \left(\frac{\Gamma(\sigma)}{\Gamma_C}\right)(1 - b) / b\right]^{-1} < 1, \] which is always satisfied since \( \Gamma_C / \Gamma(\sigma) = a[a(1 - \sigma) + \sigma b]^{-1} > 0 \). The condition of Proposition 3, which guarantees the steady state transfer paradox at the golden rule, is given by
\[
1 < \left(\frac{D\sigma}{\sigma}\right)^2 \left(\frac{a}{1 - b}\right)\frac{\Gamma_C}{\Gamma(\sigma)} \tag{6.1}
\]
By inspection, the right hand inequality is more likely satisfied when \( a + b > 1 \) and \( a > b \), that is, when the consumption good is capital-intensive.

Following Cremers (2001), factor price equalization occurs at the initial steady state if the ratios \( \sigma^D / \sigma \) and \( \sigma^R / \sigma \) fall in a range determined by \( \Gamma_C / \Gamma(\sigma) \) and \( \Gamma_I / \Gamma(\sigma) \), where
\[\Gamma_I / \Gamma(\sigma) = [(b(1 - a)) / (a(1 - b))] a[(1 - \sigma)a + \sigma b]^{-1} \] and \( \Gamma_C > \Gamma_I \) for \( a > b \) and vice versa.\(^{12}\)

This imposed similarity in savings rates prevents the country-specific steady state capital-labor ratios from lying outside the steady state cone of diversification. By comparison with (6.1), the analytical requirements for factor price equalization are not in apparent contradiction to those for a transfer paradox at the golden rule.

Away from the golden rule, the donor experiences a (paradoxical) steady state welfare improvement if

\(^{12}\) See Kehoe and Bajona (2006a), (2006b) for discussions related to factor price equalization when countries have identical rates of time preference.
\[ 1 < \left( \frac{D_\sigma}{\sigma} \right)^2 \left\{ \frac{a}{1 - b} \right\} \Gamma_C - \frac{(1 - \rho) \sigma^D}{\sigma} D_\sigma \left\{ \frac{1 - a}{1 - b} \right\} \Gamma_C \] (6.2)

where \( 1 - \rho = \frac{\sigma(1-b)-a}{\sigma(1-a)} \). \(^{13}\)

Next, a numerical example is presented to demonstrate that parameter values exist that verify factor price equalization and a donor’s welfare improvement in and out of the steady state.\(^{14}\) The parameters are restricted as follows: \( a = 0.99 \), \( b = 0.3 \), \( \sigma^D = 0.25 \) and \( \sigma = 0.55 \). These values imply that \( \theta_{KC} = 0.99 \), \( \theta_{KI} = 0.3 \), \( \theta_{LI} = 0.7 \), \( \theta_{LC} = 0.01 \), \( \lambda_{LC} = 0.615 \), \( \lambda_{KC} = 0.997 \), \( \lambda_{LI} = 0.385 \), and \( \lambda_{KI} = 0.003 \) and \( D_\sigma = -0.6 < 0 \). Thus, \( \sigma^D/\sigma = 0.455 \) and falls within the required range for steady state factor price equalization\([0.007, 1.622]\). The steady state intertemporal terms of trade effect equals 0.35 and, since positive, indicates that the initial equilibrium is below the golden rule. The overall welfare effect is \( 1.03 > 1 = \varepsilon \), indicating a steady state welfare improvement for the donor. Finally, the donor’s welfare improves away from the steady state as well, since at date \( t = 4 \) our numerical values determine that the left-hand term in (5.3) is equal to \( 1.009 > 1 \); this value increases at each subsequent date until it converges to the steady state value of 1.03.

7. Conclusion

This paper explores the welfare implications of permanent international transfers in a two-sector overlapping generations model. Within this framework, it has been possible to

\(^{13}\) The analytical expression for donor’s welfare at each date along the transition path does not simplify greatly with the adoption of specific functional forms and hence is omitted here, though it is still considered in the numerical analysis.
provide an analysis that incorporates both the static effects described by the early trade theoretic literature and also the dynamic effects explored by dynamic one-sector models. It is demonstrated that the effects of an international transfer on the static terms of trade always work in favor of a steady state transfer paradox, though elasticities of substitution can neither be too large nor too small for the transfer paradox to arise. Moreover, neither Walrasian nor dynamic stability are sufficient to rule out the possibility of a transfer paradox, in contrast to results from static analyses. Dynamics imply an additional intertemporal terms of trade effect which has a uniform influence on the welfare of both donor and recipient in the steady state, thus making it possible for both to experience a welfare improvement. It is also possible for the donor to experience a paradoxical welfare improvement in the transitional periods.

Appendix

To show that \( \hat{\eta}_w = \frac{\hat{w}}{\hat{p}} = \frac{-\theta_{KC}}{\Delta} \) and \( \hat{\eta}_r = \frac{\hat{r}}{\hat{p}} = \frac{\theta_{LC}}{\Delta} \), differentiate the pricing equations to get

\[
\begin{align*}
\theta_{LC} \hat{w} + \theta_{KC} \hat{r} &= 0, \\
\theta_{LI} \hat{w} + \theta_{KI} \hat{r} &= \hat{p},
\end{align*}
\]

or,

\[
\begin{bmatrix}
\theta_{LC} & \theta_{KC} \\
\theta_{LI} & \theta_{KI}
\end{bmatrix}
\begin{bmatrix}
\hat{w} \\
\hat{r}
\end{bmatrix} = \begin{bmatrix}
0 \\
\hat{p}
\end{bmatrix}
\]

where \( \theta_{ij} \) is factor \( i \)'s cost share of good \( j \), \( \theta_{LC} = a_{LC} \hat{w}; \theta_{LI} = a_{LI} \hat{w}/\hat{p}; \theta_{KC} = a_{KC} \hat{r}; \theta_{KI} = a_{KI} \hat{r}/\hat{p} \) and \( \sum \theta_{ij} = 1 \). Then, using Cramers’ rule, \( \eta_w \equiv (\hat{w}/\hat{p}) = -\theta_{KC}/\Delta \) and \( \eta_r \equiv \hat{r}/\hat{p} = \theta_{LC}/\Delta \), where, making use of

\[
\sum \theta_{ij} = 1, \quad \Delta \equiv \theta_{LC} - \theta_{LI} = \theta_{KI} - \theta_{KC}
\]

and where \( \Delta > (\leq) 0 \) as the consumption good is respectively labor- or capital-intensive.

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\(^{14}\) The same parameter values can be used to demonstrate that the steady state welfare of the recipient also increases, implying that the transfer is Pareto-improving, as would be expected given Proposition 4.
**Proof of Lemma 1** Walrasian stability requires that world per capita excess demand for investment goods is decreasing in $p$ at the initial equilibrium. At the initial equilibrium, (3.2) is written $\sigma \frac{w}{p_t} = I(p_t, k_t)$ so that we require $\partial \left( \frac{\sigma w}{p_t} \right) / \partial p_t - \partial I(p_t, k_t) / \partial p_t < 0$. The sufficient condition then follows since $\partial \left( \frac{\sigma w}{p_t} \right) / \partial p_t = (\sigma / p)dw / dp - \sigma w / p^2 = (\sigma w / p^2)(\eta_{wp} - 1)$ and $\partial I(p_t, k_t) / \partial p_t = \left( \frac{1}{p} \right) \eta_{lp} = \left( \sigma w / p^2 \right) \eta_{lp}$.

**Proof of Lemma 2** With full depreciation, $k_{t+1} = I_t$ at the initial equilibrium. So, $\hat{k}_{t+1} = \hat{I}_t = \eta_{lp} \hat{p}_t + \eta_{lk} \hat{k}_t$. The world capital market-clearing condition further implies $\hat{w}_t = \hat{p}_t + \hat{I}_t$. Substituting into the previous equation gives $\hat{k}_{t+1} = \eta_{lk}(\eta_{wp} - 1)\left[ \eta_{wp} - 1 - \eta_{lp} \right]^{-1} \hat{k}_t$ and therefore dynamic stability requires $\left| \eta_{lk}(\eta_{wp} - 1)\left[ \eta_{wp} - 1 - \eta_{lp} \right]^{-1} \right| < 1$. The first term, $\eta_{lk}(\eta_{wp} - 1)$, is negative regardless of the factor intensity assumption and Walrasian stability implies that the second term is negative. Thus, the product is positive which enables the dynamic stability condition to be expressed by the sufficient condition of the lemma. □

**References**


